# AN INTRODUCTION TO QUANTUM OPTICS...

...the light as you've never seen before...

Optics and Photonics
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Rafael S. Zola

"*If things of sight such heavens be, what heavens are those we cannot see?*"

Andrew Marvell

# INTRODUCTION

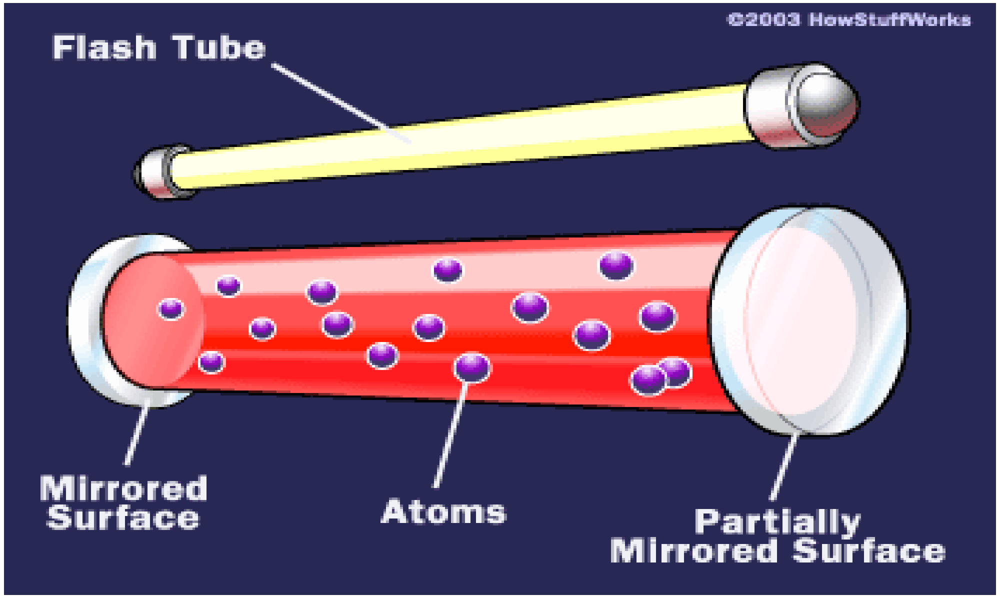
- First steps:
- Quantum by Plank;
- Photon by Einstein;

*Newton X Huygens*

- Feynman “ideal” experiment;
- And then, for the next 55 years...

# INTRODUCTION

- And the Light Amplification by Stimulated Emission of Radiation arrives:




Source:http://science.howstuffworks.com/laser5.htm

# DEFINITION

- Quantum Optics:

  *“Quantum optics is a field in quantum physics, dealing with the application of quantum mechanics to phenomena involving light and its interactions with matter.”*

# QUANTUM OPTICS OPERATORS

- Light is described in terms of field operators for creation and annihilation of photons:

$$a^\dagger = \frac{1}{\sqrt{2}}(q - ip)$$ *creation operator* or *raising operator*

$$a = \frac{1}{\sqrt{2}}(q + ip)$$ *annihilation operator* or *lowering operator*

$$H = \hbar\omega\left(a^\dagger a + \frac{1}{2}\right)$$

# QUANTUM FIELDS

- The usual description of the classical fields are given by quantum versions:

$$\hat{E}_x(z,t) = \mathcal{E}_0(a + a^\dagger)sin(kz)$$

$$\hat{B}_y(z,t) = \mathcal{B}_0\frac{1}{i}(a - a^\dagger)cos(kz)$$

where $\mathcal{E}_0$ and $\mathcal{B}_0$ are the electric and magnetic field "per photon".

# QUANTUM FIELDS

- The number state has well-defined energy but not well defined electric field:

$$\langle n|\hat{E}_x(z,t)|n\rangle = \mathcal{E}_0 sin(kz)\left[\langle n|a|n\rangle + \langle n|a^\dagger|n\rangle\right] = 0$$

but the fluctuations of the electric field are not zero:

$$\Delta E_x = \sqrt{2\mathcal{E}_0} sin(kz)\left(n + \frac{1}{2}\right)^{1/2}$$

# THE QUANTUM PHASE

- The single mode electric field is:

$$\hat{E}_x(r,t) = i\left(\frac{\hbar\omega}{2\varepsilon_0 V}\right)^{1/2} e_x(ae^{ik\cdot r - i\omega t} + a^\dagger e^{-ik\cdot r + i\omega t})$$

the classical theory:

$$\hat{E}_x(r,t) = \frac{1}{2}E_0 e_x(e^{ik\cdot r - i\omega t + \phi} + e^{-ik\cdot r - i\omega t + \phi})$$

May $a$ and $a^\dagger$ have information about the phase?

# THE QUANTUM PHASE

- The Susskind-Glogower (SG) operators:

$$\hat{E} = (aa^\dagger)^{-1/2}a \quad \text{and} \quad \hat{E}^\dagger = a^\dagger(aa^\dagger)^{-1/2}$$

are the quantum phase factors with eigenstates:

$$\hat{E}|\phi\rangle = e^{i\phi}|\phi\rangle$$

$$|\phi\rangle = \sum_{n=0}^{\infty} e^{in\phi}|n\rangle$$

# COHERENT STATE – THE MOST CLASSICAL QUANTUM STATE

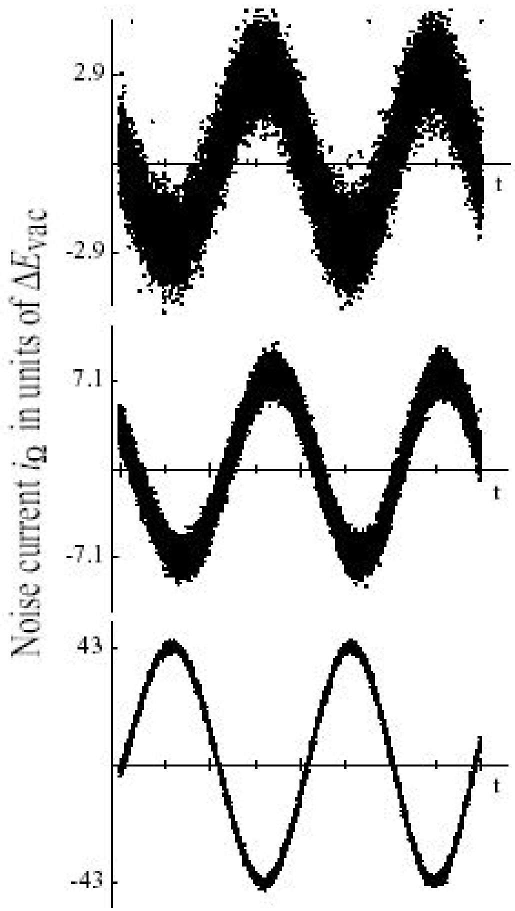
- The expected value of the "classical" electric field is not zero – single states are not the "real" description. Why not define:

$$|\alpha\rangle = \sum_{n=0}^{\infty} C_n |n\rangle$$

which give us the classical expected value of the electric field. And...

$$\langle(\Delta X_1)^2\rangle_\alpha = \frac{1}{4} = \langle(\Delta X_2)^2\rangle_\alpha$$

# COHERENT STATE – THE MOST CLASSICAL QUANTUM STATE



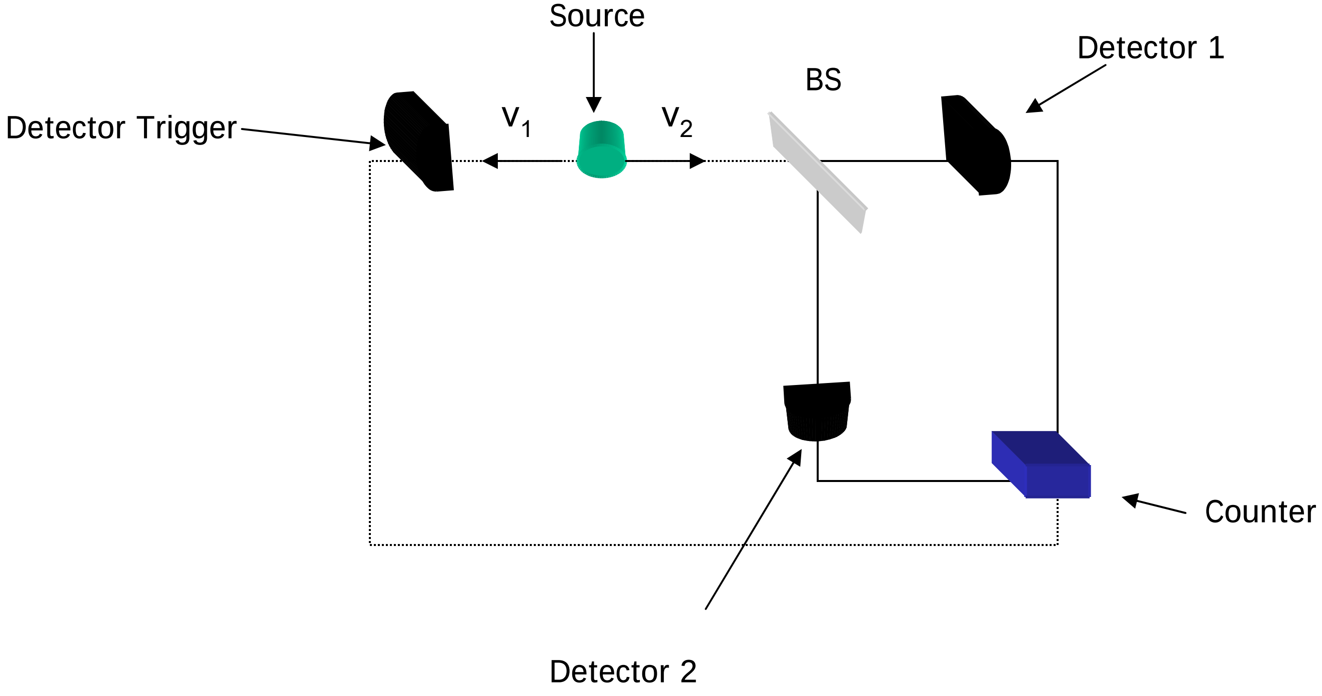

The operators “phase” and “amplitude” do not commute and the lower limit is the coherent state:

LASER

source: http://gerdbreitenbach.de/gallery/

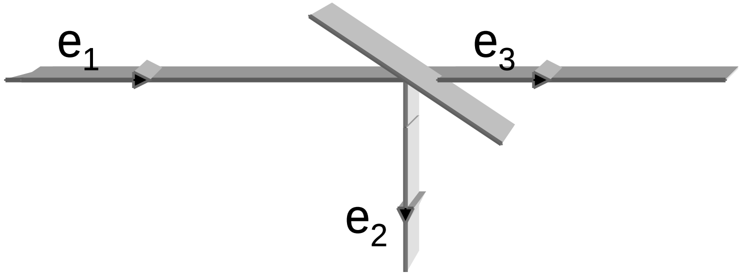
# A LITTLE BIT OPTICS – BEAM SPLITTERS AND INTERFEROMETERS

Grangier experiment:



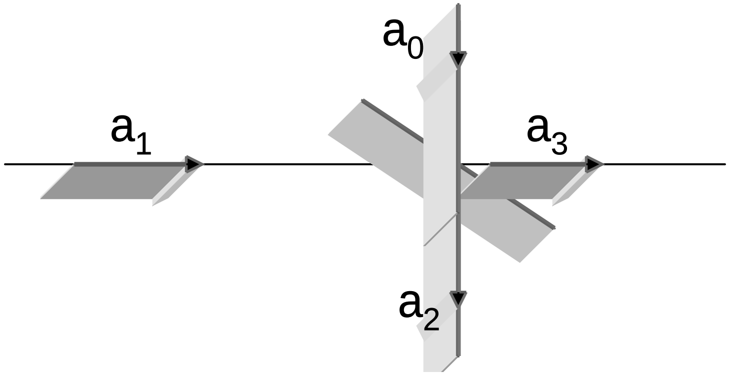
# A LITTLE BIT OPTICS – BEAM SPLITTERS AND INTERFEROMETERS

Classical beam splitter




$$e_2 = re_1, \;\; e_3 = te_1 \;\text{ and }\; |r|^2 + |t|^2 = 1$$

Quantum beam splitter




$$a_2 = ra_1 + t'a_0, \;\; a_3 = ta_1 + r'a_0$$

# A LITTLE BIT OPTICS – BEAM SPLITTERS AND INTERFEROMETERS

If we have a 50:50 beam splitter:

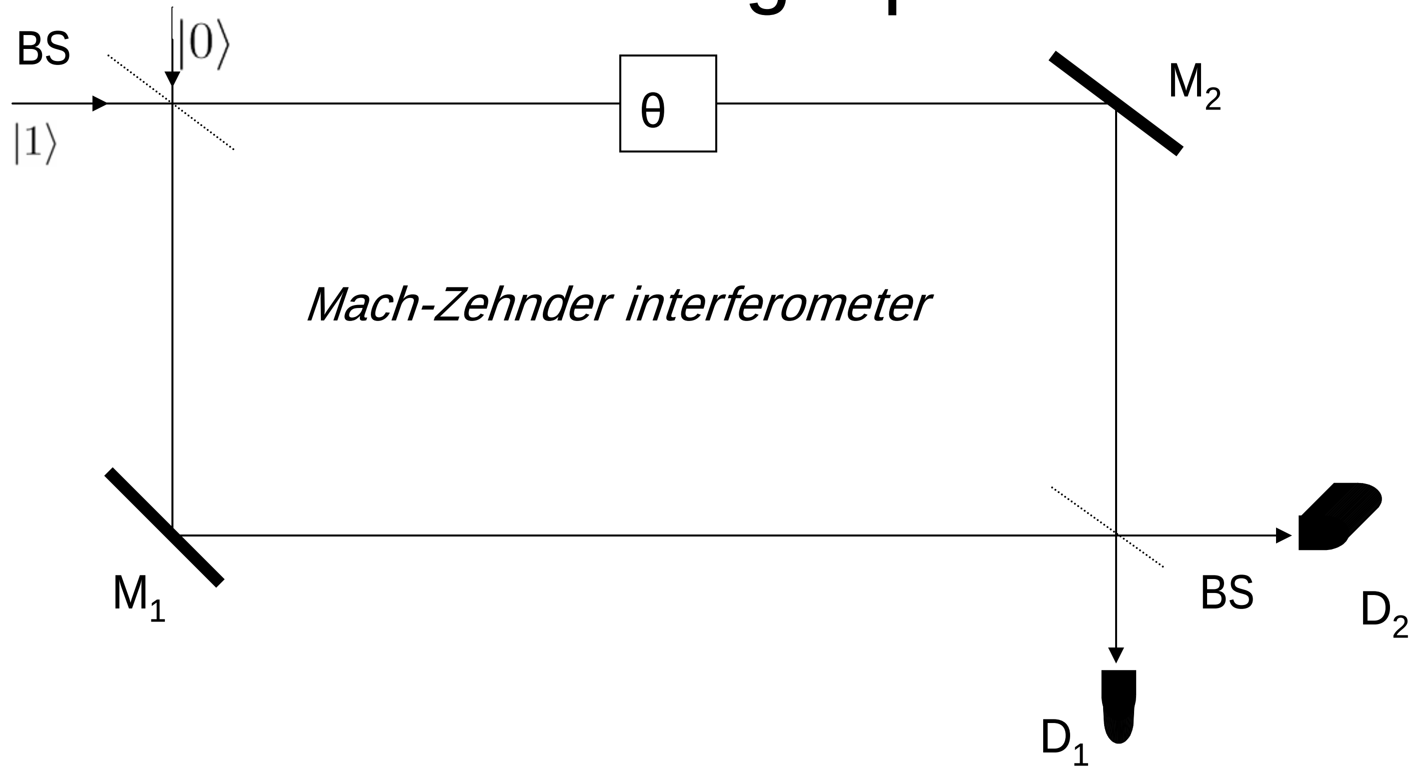
$$a_2 = \frac{1}{\sqrt{2}}(a_0 + ia_1), \ \ a_3 = \frac{1}{\sqrt{2}}(ia_0 + a_1)$$

so, given an incoming state $|0\rangle_0|1\rangle_1$

$$|0\rangle_0|1\rangle_1 \xrightarrow{\text{BS}} \frac{1}{\sqrt{2}}(i|1\rangle_2|0\rangle_3 + |0\rangle_2|1\rangle_3)$$

# A LITTLE BIT OPTICS – BEAM SPLITTERS AND INTERFEROMETERS

- Interference of single photons:



# A LITTLE BIT OPTICS – BEAM SPLITTERS AND INTERFEROMETERS

- For a incoming state $|0\rangle_0|1\rangle_1$ :

$$|0\rangle_0|1\rangle_1 \xrightarrow{\mathrm{BS}} \frac{1}{\sqrt{2}}(i|1\rangle_2|0\rangle_3 + |0\rangle_2|1\rangle_3)$$

the phase shifter is represented by

$$|\phi\rangle = \sum_{n=0}^{\infty} e^{in\phi}|n\rangle$$

# A LITTLE BIT OPTICS – BEAM SPLITTERS AND INTERFEROMETERS

so…

$$\frac{1}{\sqrt{2}}(i|1\rangle_2|0\rangle_3 + |0\rangle_2|1\rangle_3) \xrightarrow{\text{BS}} \frac{1}{\sqrt{2}}(i|1\rangle_2|0\rangle_3 + e^{i\theta}|0\rangle_2|1\rangle_3)$$

and, after the next beam splitter

$$\frac{1}{\sqrt{2}}(i|1\rangle_2|0\rangle_3 + |0\rangle_2|1\rangle_3) \xrightarrow{\text{BS}} \frac{1}{\sqrt{2}}\left[(e^{i\theta} - 1)|0\rangle_2|1\rangle_3 + i(e^{i\theta} + 1)|1\rangle_2|0\rangle_3\right]$$

# A LITTLE BIT OPTICS – BEAM SPLITTERS AND INTERFEROMETERS

then we have states with probability:

$$P_{01} = \frac{1}{2}(1 - \cos\theta)$$

$$P_{10} = \frac{1}{2}(1 + \cos\theta)$$

Nonlocality – detection without interaction!

# AND QUANTUM OPTICS...

...*The squeezed states of light (quantum noise); Optical trap; Quantum entanglement; Quantum teleportation, Single photon experiments; Interactions of single photon and matter (optical cavity); Quantum networks; Quantum communication; Bose Einstein condensates; Quantum encryption and so on...*


"The word classical means only one thing in science: it's wrong!" – *B.R. Frieden*

# REFERENCES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