1 Introduction and Ray Optics

Optics is the study of light and its interaction with matter.

Light is visible electromagnetic radiation, which transports energy and momentum (linear and angular) from source to detector.

Photonics includes the generation, transmission, modulation, amplification, frequency conversion and detection of light.

,Methods of studying light, in historical order are:
-ray optics
-wave optics
-electromagnetic optics
-photon optics (E & M fields are wavefunctions of photons).

In this course, we will focus on electromagnetic optics.

Maxwell's equations give accurate descriptions of most optical phenomena.

However, for pedagogical reasons, we begin, with ray optics.

1.1 Ray Optics

Ray optics is the simplest theory of light. Rays travel in optical media according to a set of geometrical rules; hence ray optics is also called geometrical optics.

Ray optics is an approximate theory, but describes accurately a variety of phenomena.

Ray optics is concerned with the *locations* and *directions* of light rays, which carry photons and light energy (They also carry momentum, but the direction of the momentum may be different from the ray direction). It is useful in describing image formation, the guiding of light, and energy transport.

Optical systems are often centered around an axis, called the optical axis. If rays are nearly parallel to such an axis, they are called *paraxial rays.* The study of paraxial rays is called paraxial optics.

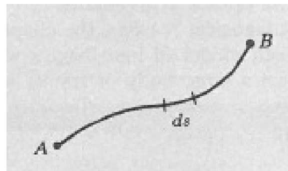
1.2 Postulates of Ray Optics

1. Light travels in the form of rays (can think of rays as photon currents). Rays are emitted by light sources, and can be observed by light detectors.

2. An optical medium (through which rays propagate) is characterized by a real scalar quantity $n \geq 1$, called the refractive index. The speed of light in vacuum is $c = 3 \times 10^8 m/s$.The speed of light in a medium is $v = c/n$; this is the definition of the refractive index. The time taken by light to cover a distance d is $t = nd/c$; it is proportional to nd, which is called the *optical path length.*

3. In an inhomogeneous medium, the refractive index $n(\mathbf{r})$ varies with position; hence the optical path length OPL between two points A and B is

$$OPL = \int_A^B n(\mathbf{r})ds$$



where ds is an element of length along the path. The time t taken by light to go from A to B is $t = OPL/c$.

4. **Fermat′sPrinciple** Light rays between the points A and B follow a path such that the time of travel, relative to neighboring paths, is an extremum (minimum). This means that the variation in the travel time, or, equivalently, in the optical path lenght, is zero. That is,

$$\delta \int_A^B n(\mathbf{r})ds = 0 \tag{1}$$


Pierre de Fermat

Usually, the extremum is a minimum; then *light rays travel along the path of least time*. If there are many paths with the minimum time, then light rays travel along all of these simultaneously.

Why should Fermat's principle work? Were is the physics? (Fermat's principle is the main principle of quantum electrodynamics! It is a consequence of Huygen's principle: waves with extremal paths contribute the most due to constructive interference.)

1.2.1 Propagation in a homogeneous medium

In a homogeneous medium, the refractive index n is the same everywhere, so is the speed of light v. Therefore the optical path length of least time is the shortest one - that is, a straight line.

Proof: Suppose the path taken by light is along the curve described by $y(x)$. The optical path length is

$$OPL = \int_A^B nds = n \int_A^B ds \tag{2}$$

We want to minimize the OPL. Suppose $y_0(x)$ is the shortest path. Then we must have that any other path is longer. Consider the path $y(x) = y_0(x) + \varepsilon(x)$, where $\varepsilon(x)$ is small. We note that

$$ds = \sqrt{dx^2 + dy^2} = dx\sqrt{1 + (\frac{dy}{dx})^2} = dx\sqrt{1 + y'^2(x)} \tag{3}$$

Since $y'(x) = y_0^{'}(x) + \varepsilon'(x)$, we can write

$$ds = dx\sqrt{1 + y_0'^2 + 2y_o'\varepsilon' + \varepsilon'^2} \tag{4}$$

If ε' is also small, we have, approximately

$$ds = dx\sqrt{1 + y_0'^2}\sqrt{1 + \frac{2y_o'\varepsilon'}{1 + y_0'^2}} \simeq dx\sqrt{1 + y_0'^2} + dx\frac{\varepsilon' y_0'}{\sqrt{1 + y_0'^2}} \tag{5}$$

and the optical path length which we varied, OPL_v is

$$OPL_v = n\int_A^B (\sqrt{1 + y_0'^2} + \frac{\varepsilon' y_0'}{\sqrt{1 + y_0'^2}})dx \tag{6}$$

Now we said that $y_0(x)$ is the shortest path, that is, the shortest opical path length OPL_S

$$OPL_S = n\int_A^B (\sqrt{1 + y_0'^2})dx \tag{7}$$

is a minimum. This means that OPL_v cannot be less than OPL_S.

If we could choose ε' freely, then we could argue that the coefficient of ε' must be equal to zero, otherwise, by choosing ε' appopriately, we chould make OPL_v less than OPL_S, in violation of our original assumption. However, we cannot choose ε' freely, since we have the constraint that $\varepsilon(x) = 0$ at the end points. (This is because all paths must go through the end points, and since $y_0(x)$ certainly does, $\varepsilon'(x)$ must vanish there.)

We therefore recall that

$$\int_A^B udv = uv|_A^B - \int_A^B vdu \tag{8}$$

and write

$$\int_A^B \frac{y_0'}{\sqrt{1 + y_0'^2}}\varepsilon' dx = \frac{y_0'}{\sqrt{1 + y_0'^2}}\varepsilon|_A^B - \int_A^B \varepsilon\frac{d}{dx}(\frac{y_0'}{\sqrt{1 + y_0'^2}}) \tag{9}$$

Since ε vanishes at the end points, the first term on the rhs is zero, and we have

A

$$OPL_v = n\int_A^B (\sqrt{1 + y_0'^2} - n\int_A^B \varepsilon\frac{d}{dx}(\frac{y_0'}{\sqrt{1 + y_0'^2}}) \tag{10}$$

Now we can argue that if the coefficient of ε is not zero, then, by choosing $\varepsilon(x)$ appropriately, we can make OPL_v less than OPL_S, in violation of our original assumption. We must therefore have

$$\frac{d}{dx}\left(\frac{y_0'}{\sqrt{1+y_0'^2}}\right)=0 \tag{11}$$

or

$$\frac{y_0'}{\sqrt{1+y_0'^2}}=C \tag{12}$$

or

$$y_0'=m \tag{13}$$

and

$$y_0=mx+b \tag{14}$$

So if $y_0(x)$ is the shortest path, giving the minimum OPL, we must have

$$y(x)=mx+b \tag{15}$$

and the path must be a straight line.

This illustrates the method for finding the function which makes the variation vanish - it is the basic idea of variational calculus.

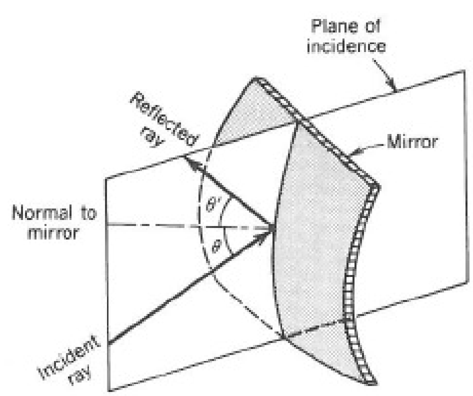
We conclude therefore that *in a homogeneous medium, light rays travel in straight lines.*

The principle that light in a homogeneous medim takes the shortest path is Hero's Principle

1.2.2 Reflection from a mirror

Mirrors are madefrom metallic surfaces, or dielectric films, or other highly reflective surfaces.

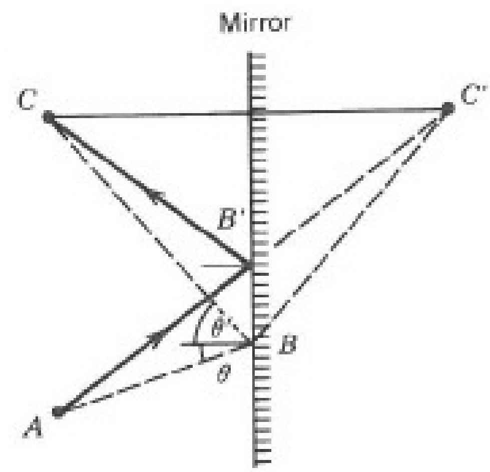
A reflective surfaces is shown below.



The plane of incidence is defined by the surface normal and the incident ray. (Note: the plane of incidence may be different when we talk about waves!)

The angle θ is the angle of incidence, and θ' is the angle of reflection.

Consider the plane mirror shown below.

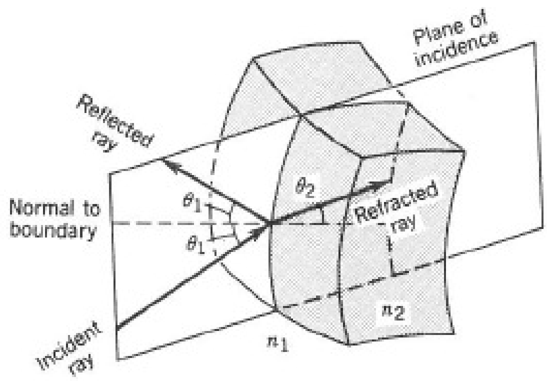


According to Fermat's principle, B must be such that the $OPL = AB + BC$ is a minimum. (Assume there is no delay associated with the reflection) If C and C' lie on the normal to the mirror, and are equidistant from it,. then it is clear that $AB'C'$ is the shortest distance between A and C'. Since $B'C = B'C$, it is clear that $AB'C$ is the shortest OPL. This means that $\theta = \theta'$.

The reflected ray lies in the plane of incidence; the angle of reflection is equal to the angle of incidence.

1.2.3 Reflection and Refracton ant the boundary between two media

At the boundary between two media with refractive indices n_1 and n_2, the incident ray is split into a reflected ray, and a refracted (transmitted) ray.



The reflected ray obeys the law of reflection.

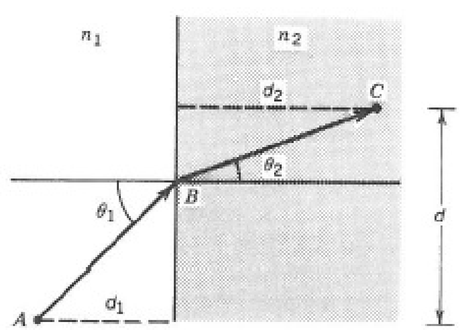
The refracted ray obeys the law of refraction:

The refracted ray lies in the plane of incidence; theangle of refraction θ_2 is related to the angle of incidence θ_1 by Snell′s Law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Proof:

Consider



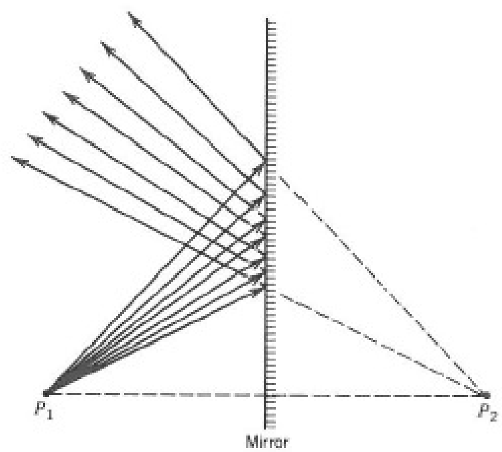
This will be a homework assignment.

1.3 Simple Optical Components

These can be understood in terms of propagation in straight lines (in homogeneous media) and the laws of reflection and refraction.

1.3.1 Planar Mirror

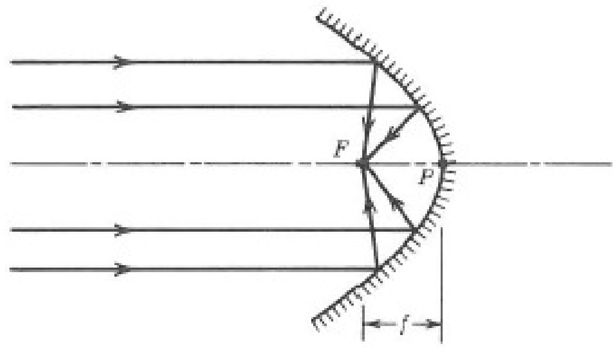
Reflects light originating from point P_1 such that the reflected rays appear to originate from a point P_2 behind the mirror. P_2 is called the image of P_1.



Planar Mirror

1.3.2 Paraboloidal Mirror

Surface is paraboloid of revolution; it focuses all incident rays parallel to its axis to a single point called the focus. the distance PF is the focal length f. They are often used as light-collecting elements in telescopes, and reflectors to make parallel beams in flashlights.



Parabolic Mirror

(Recall: A parabola is the set of points equidistant from a point (focus) and a straight line.) Time to reach focus is the same for all rays. If $y = x^2$, what is f?

1.3.3 Image formation by Parabolic Mirror

We now consider image formation by a parabolic mirror.

When we see an object, we 'see' light coming to our eyes from each point on the surface of the object. The light can originate in the object (hot filament of a light bulg), or it can be reflected light when the object is illumated by outside sources.

Imagine placing an object on axis (shown by arrow of height y_1 in the Figure below), and we consider paths of rays of light leaving the object.

Light rays will be leaving the object, and will be reflected by the mirror.

Consider the ray travelling horizontally from the tip to the mirror, reflected through angle β,and passing through the focus.

Consider next the ray travelling from the tip to the focus,passing through the focus, continuing to the mirror, then reflected through the angle α and travelling horizontally away from the mirror.

These two rays come together at the tip of the image (shown by the arrow of height y_2)

Consider a third ray, showed by the dashed line, travelling from the tip of the object to point where the mirror and the axis intersect. It will be reflected, and also pass through the point at the tip of the image.

In this way, the parabolic mirror collects the rays from the tip of the object, and collects them at th tip of the image. The same is true of every other point on the object and image. This is image formation.

We are interested in knowing where the image will be, and how large will it be.

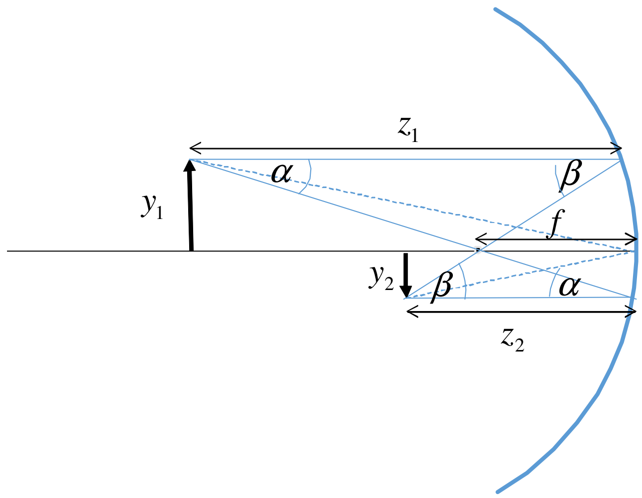


Figure 1: Imaging with a paraboloidal mirror

First, we consider the location.

We begin by noting that

$$(z_1 - f)\tan\alpha = y_1$$

and

$$(z_2 - f)\tan\beta = y_2$$

We also note that

$$y_1 = f\tan\beta$$

and

$$y_2 = f\tan\alpha$$

Substituting for y_1 and y_2, we get

$$(z_1 - f)\tan\alpha = f\tan\beta$$

and

$$(z_2 - f)\tan\beta = r\tan\alpha$$

Multiplying these gives

$$(z_1 - f)(z_2 - f) = f^2$$

and simplifying, we get

$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f}$$

This is exact; it is an important general result.

Note that z is measured from the mirror, both image and object distances must be greater than the focal length f. The image is inverted.

The magnification is defined as

$$M = \frac{y_2}{y1}$$

Since the triangles above and below the axis are similar, we immediately have

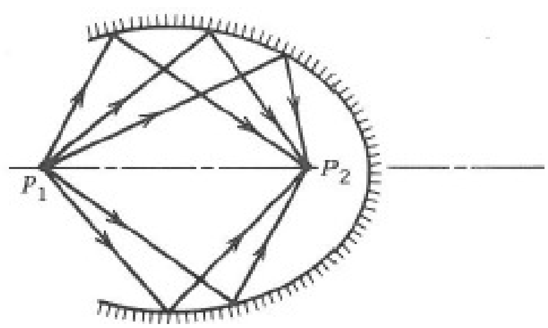
$$M = \frac{z_2}{z_1}$$

This is another important general result.

So if the location of the object is known, the location and size of the image can be immediately determined.

1.3.4 Elliptical Mirrors

All rays emitted from one of two foci, say P_1, are focussed (imaged) at the other, P_2.



Elliptical mirror.

(Recall: An ellipse is the set of points such that the sum of the distance from two points (foci) is constant.)

Spherical Mirrors Does not focus in general, however, for paraxial rays, a spherical mirror with radius R it resembles a parabolic mirror with focus $f = R/2$.

All paraxial rays from point P_1 on axis are focused to point P_2 on axis. Now (ignoring signs in the illustration)

$$\theta_1 = \theta_0 - \theta \tag{16}$$

and

$$\theta_2 = \theta_0 + \theta \tag{17}$$

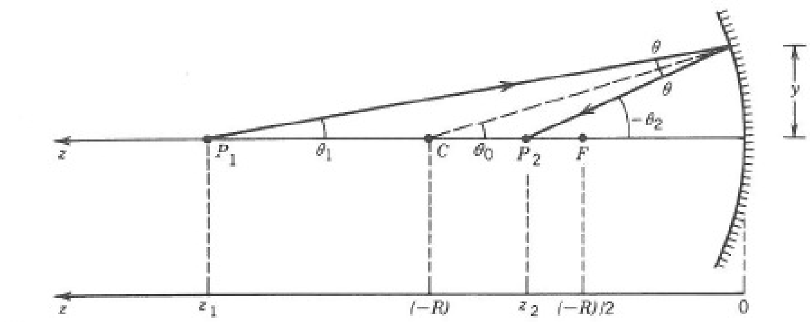


Figure 2: Spherical mirror in paraxial approximation

so

$$\theta_1 + \theta_2 = 2\theta_0 \tag{18}$$

Since for small angles $\alpha \simeq \tan\alpha$, we can write the above as

$$\tan\theta_1 + \tan\theta_2 = 2\tan\theta_0 \tag{19}$$

and, to a good approximation

$$\frac{y}{z_1} + \frac{y}{z_2} = 2\frac{y}{R} \tag{20}$$

and

$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{2}{R} = \frac{1}{f} \tag{21}$$

1.3.5 Imaging by a spherical mirror

Spherical mirrors form images the way lenses do.

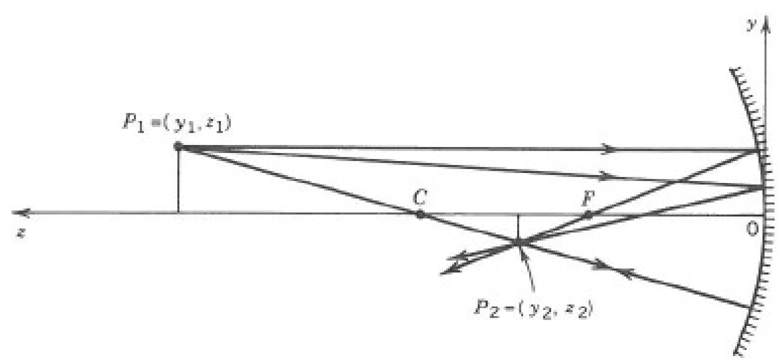


Image formation by spherical mirror

Here, again,

$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f} \tag{22}$$

The magnification is

$$M = \frac{y_2}{y_1} = \frac{z_2}{z1} \tag{23}$$

(consider ray from P_1 to 0, and it's reflection, which also goes through P_2).

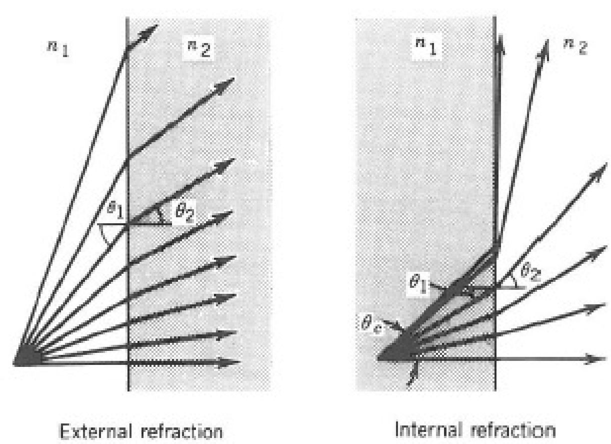
1.4 Planar Boundaries

Use Snell's Law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{24}$$

for small angles,

$$n_1\theta_1 \simeq n_2 \sin\theta_2$$

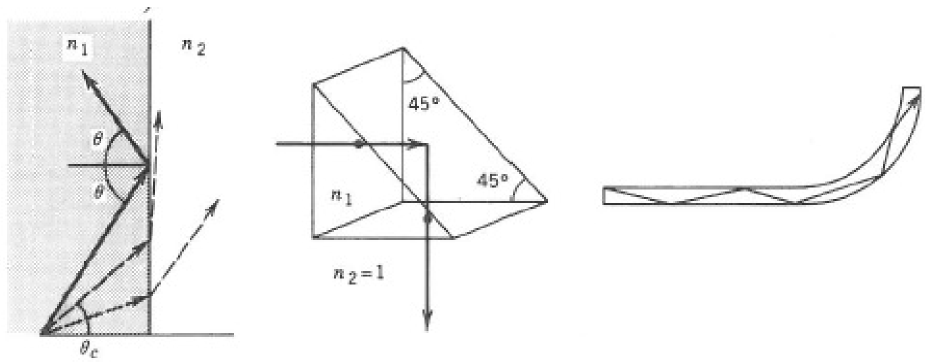


Refraction by planar boundary

Total internal reflectiion (TIR): $\theta_2 = \pi/2$; then

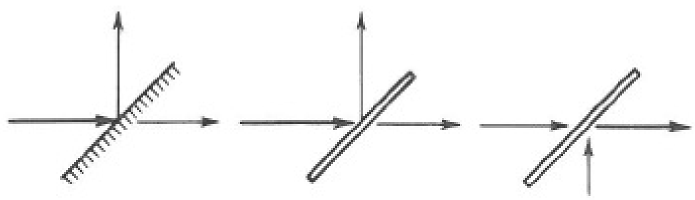
$$\theta_{1c} = \sin^{-1}\frac{n_2}{n_1} \tag{25}$$

For $\theta_1 \geq \theta_{1c}$, all the light is reflected. Examples are shown below.



TIR at planar boundary; reflecting prism and optic fiber

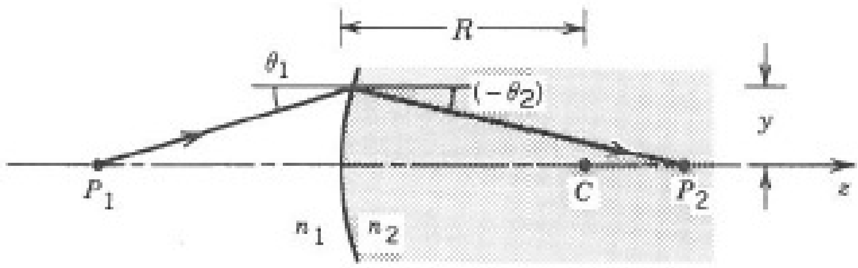
1.4.1 Beamsplitters & Beam combiners



Partially reflecting mirror, thin glass plate (splitter and combiner)

1.5 Spherical Boundaries and Lenses

Consider the spherical interface below:



Spherical interface

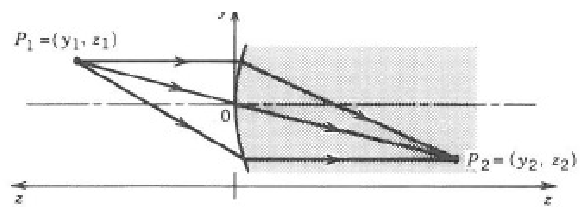


Figure 3: Image formation

It is straightforward to show that (ignoring signs in diagram) for paraxial rays,

$$n_1\theta_1 + n_2\theta_2 = (n_2 - n_1)\frac{y}{R} \tag{26}$$

and

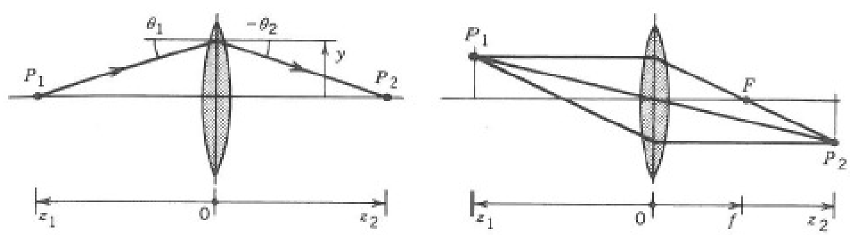
$$\frac{n_1}{z_1} + \frac{n_2}{z_2} = \frac{n_2 - n_1}{R} \tag{27}$$

and there is image formation, as for a spherical mirror:The magnification is

$$M = \frac{y_2}{y_1} = \frac{z_2}{z_1} \tag{28}$$

1.5.1 Thin Lenses

The surfaces of thin lenses may be thought of as the intersection of two spheres, with radii R_1 and R_2.



Ray bending and image formation by a thin lens.

It is straightforward to show that, for paraxial rays,

$$\theta_1 + \theta_2 = \frac{y}{f} \tag{29}$$

where

$$f = (n-1)(\frac{1}{R_1} + \frac{1}{R_2}) \tag{30}$$

and

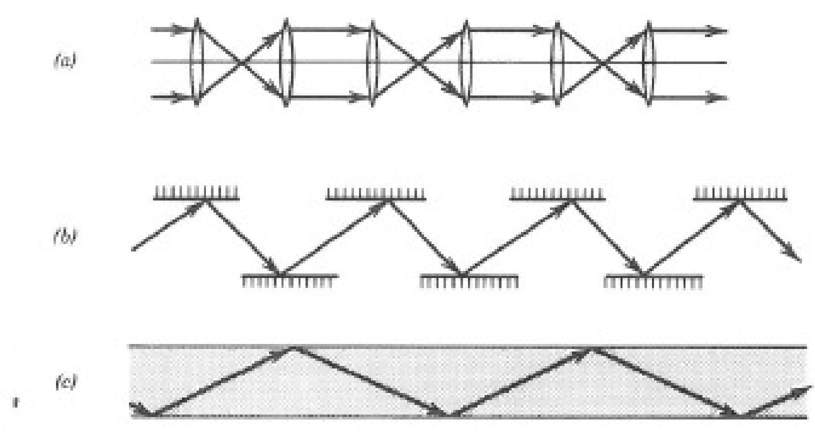
$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f} \tag{31}$$

and

$$M = \frac{y_2}{y_1} = \frac{z_2}{z_1} \tag{32}$$

1.5.2 Light Guides

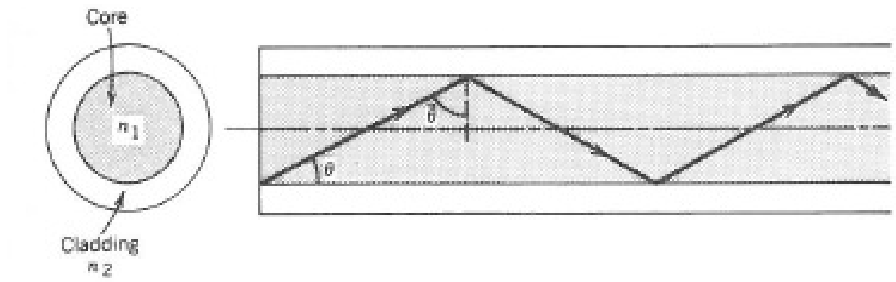
Light may be quided from one location to another in a variety of ways.
Some are shown below.



Schemes for guiding light

Lenses are partially reflective and mirrors are partially absorptive, a. and b are not particularly efficient. Total internal reflection (TIR) is efficient and inexpensive. Optic fibers rely on TIR.

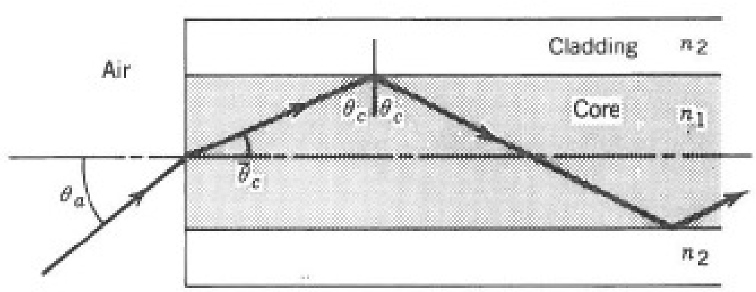
A stepped index fiber is shown below.



Stepped index optical fiber utilizing TIR

1.5.3 Numerical Aperture and Angle of Acceptance of Optical Fiber

For propagation via TIR, the angle of incidence between the core and cladding must be no greater than the critical angle for TIR.



Acceptance angle of optical fiber

If the angle of incidence is

$$n_0 \sin\theta_a = n_1 \sin\theta_t \tag{33}$$

and we must have

$$n_1 \sin\theta_c = n_2 \tag{34}$$

or

$$\sin\theta_c = \frac{n_2}{n_0}$$

We also have

$$\theta_t + \theta_c = \frac{\pi}{2} \tag{35}$$

We would like to eliminate θ_c and θ_t, and have an expression for θ_a. We write

$$\cos(\theta_t + \theta_c) = \cos\theta_t \cos\theta_c - \sin\theta_t \sin\theta_c = 0 \tag{36}$$

and

$$\tan\theta_t = \frac{1}{\tan\theta_c} \tag{37}$$

Now

$$n_0 \sin\theta_a = n_1 \frac{\tan\theta_t}{\sqrt{1+\tan^2\theta_t}} = n_1 \frac{1}{\sqrt{1+\cancel{t}an^2\theta_c}} = n_1 \cos\theta_c = n_1\sqrt{1-\frac{n_2^2}{n_1^2}} \tag{38}$$

and finally

$$n_0 \sin\theta_a = \sqrt{n_1^2 - n_2^2} \tag{39}$$

The quantity $n_0 \sin\theta_a$ is known as the numerical aperture NA of the fiber.

$$NA = n_0 \sin\theta_a = \sqrt{n_1^2 - n_2^2} \tag{40}$$

The angle θ_a is the angle of acceptance; rays with angles of incidence greater than θ_a will not undergo TIR, and will not propagate.

1.6 Graded Index (GRIN) Optics

Fermat's principle tells us that light takes the path of least time; hence the optical path lenght is an extremum. That is,

$$OPL = \int_A^B n(\mathbf{r})ds \tag{41}$$

is an extremum. Let us see what this tells us about ray propagation.

Suppose that the location of points on the path the ray follows is given by the vector $\mathbf{r}(s)$ from an arbitrary reference point in space. The quantity s is distance measured along the path, from A to B. Now

$$\sqrt{d\mathbf{r}^2} = ds \tag{42}$$

and we can write

$$OPL = \int_A^B n(\mathbf{r})\sqrt{(\frac{\partial \mathbf{r}}{\partial s})^2} ds \tag{43}$$

Let us suppose that the path which gives the extremum of the OPL is $\mathbf{r}_0(s)$; s is the distance measured along the path $\mathbf{r}_0(s)$. Clearly $(\frac{\partial \mathbf{r}_0}{\partial s})^2 = 1$. Now suppose we consider a small perturbation $\varepsilon(s)$ to the path; s is still measured along the path $\mathbf{r}_0(s)$. Now the new path will be

$$\mathbf{r}(s) = \mathbf{r}_0(s) + \boldsymbol{\varepsilon}(s) \tag{44}$$

(s is still measured as before, along $\mathbf{r}_0(s)$.) The OPL along this new path will be

$$OPL = \int_A^B n(\mathbf{r}_0 + \boldsymbol{\varepsilon})\sqrt{(\frac{\partial \mathbf{r}_0}{\partial s} + \frac{\partial \boldsymbol{\varepsilon}}{\partial s})^2} ds \tag{45}$$

Now we do Taylor's series expansion of both terms in the integral

$$OPL = \int_A^B [n(\mathbf{r}_0) + \nabla n(\mathbf{r}_0) \cdot \boldsymbol{\varepsilon})][\sqrt{(\frac{\partial \mathbf{r}_0}{\partial s})^2} + \frac{\frac{\partial \mathbf{r}_0}{\partial s} \cdot \frac{\partial \boldsymbol{\varepsilon}}{\partial s}}{\sqrt{(\frac{\partial \mathbf{r}_0}{\partial s})^2}}] ds \tag{46}$$

We recall that $(\frac{\partial \mathbf{r}_0}{\partial s})^2 = 1$, and, rearranging, we get

$$OPL = \int_A^B n(\mathbf{r}_0) ds + \int_A^B (\nabla n(\mathbf{r}_0) \cdot \boldsymbol{\varepsilon} + n(\mathbf{r}_0) \frac{\partial \mathbf{r}_0}{\partial s} \cdot \frac{\partial \boldsymbol{\varepsilon}}{\partial s}) ds \tag{47}$$

Now if the extremal path is $\mathbf{r}_0(s)$, then we must guarantee that the second integral vanishes - otherwise $\mathbf{r}_0(s)$ would not be the extremal path.

As before, we integrate by parts, then

$$\int_A^B n(\mathbf{r}_0) \frac{\partial \mathbf{r}_0}{\partial s} \cdot \frac{\partial \boldsymbol{\varepsilon}}{\partial s} ds = n(\mathbf{r}_0) \frac{\partial \mathbf{r}_0}{\partial s} \cdot \boldsymbol{\varepsilon}|_A^B - \int_A^B \boldsymbol{\varepsilon} \cdot \frac{\partial}{\partial s} n(\mathbf{r}_0) \frac{\partial \mathbf{r}_0}{\partial s} ds \tag{48}$$

Since $\boldsymbol{\varepsilon}$ vanishes at the points A and B, we have

$$OPL = \int_A^B n(\mathbf{r}_0) ds + \int_A^B \boldsymbol{\varepsilon} \cdot (\nabla n(\mathbf{r}_0) - \frac{\partial}{\partial s} n(\mathbf{r}_0) \frac{\partial \mathbf{r}_0}{\partial s}) ds \tag{49}$$

and we must have

$$\nabla n(\mathbf{r}_0) - \frac{\partial}{\partial s} n(\mathbf{r}_0) \frac{\partial \mathbf{r}_0}{\partial s} = 0 \tag{50}$$

and so in order to extremize the OPL, the ray must follow the path $\mathbf{r}(s)$ satisfying

$$\frac{\partial}{\partial s} n(\mathbf{r}) \frac{\partial \mathbf{r}}{\partial s} = \nabla n \tag{51}$$

This is the Ray Equation. If the refractive index profile and the location and direction of the ray is known, the Ray Equation can be solved to determine how the ray propagates. This is a very important and powerful result. (Valid only in isotropic media!)

1.6.1 Paraxial approximation of the Ray Equation

If the rays are nearly on axis (z-direction), we can let $s \simeq z$; then the Ray Equation becomes

$$\frac{\partial}{\partial z} n \frac{\partial y(z)}{\partial z} = \frac{\partial n}{\partial y} \tag{52}$$

and

$$\frac{\partial}{\partial z} n \frac{\partial x(z)}{\partial z} = \frac{\partial n}{\partial x} \tag{53}$$

Usually, this is easier to solve than the general equation Eq. 51.

Graded Index Slab If the refractive index $n = n(y)$ depends only on y, then we have, in the paraxial approximation,

$$\frac{\partial^2 y}{\partial z^2} = \frac{1}{n}\frac{\partial n}{\partial y} \tag{54}$$

This is often quite straightforward to solve.

Slab with parabolic index profile Suppose that the refractive index $n(y)$ has the form

$$n = n_0(1 - \frac{1}{2}\alpha^2 y^2) \tag{55}$$

where $\alpha y << 1$ in the regions of interest. Materials with such a profile have the trade name of SELFOC. We have

$$\frac{\partial n}{\partial y} = -n_0 \alpha^2 y \tag{56}$$

and

$$\frac{1}{n}\frac{\partial n}{\partial y} = \frac{-n_0 \alpha^2 y}{n_0(1 - \alpha^2 y^2)} \simeq -\alpha^2 y \tag{57}$$

and

$$\frac{\partial^2 y}{\partial z^2} = -\alpha^2 y \tag{58}$$

Solution:

$$y = A \cos \alpha z + B \sin \alpha z \tag{59}$$

Now

$$y(0) = A \tag{60}$$

and

$$y'(0) = \alpha B \tag{61}$$

so we can write the solution as

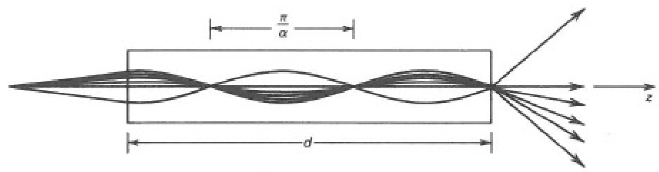
$$y = y_0 \cos \alpha z + \frac{\theta_0}{\alpha} \sin \alpha z \tag{62}$$

where $y_0 = y(0)$ and $\theta_0 = y'(0)$. The ray oscillates about the optic axis, the pitch is

$$p = \frac{\pi}{\alpha} \tag{63}$$

and the amplitude is

$$y_M = \sqrt{y_0^2 + (\frac{\theta_0}{\alpha})^2} \tag{64}$$



Ray trajectories in SELFOC slab

SELFOC cylinder - a GRIN optical fiber In the cylidrical geometry, the index is given by

$$n(x,y) = n_0(1 - \frac{1}{2}\alpha^2(x^2+y^2)) \tag{65}$$

In this case, we have

$$\frac{\partial y^2}{\partial z^2} = \frac{1}{n}\frac{\partial n}{\partial y} \simeq -\alpha^2 y \tag{66}$$

and

$$\frac{\partial x^2}{\partial z^2} = \frac{1}{n}\frac{\partial n}{\partial x} \simeq -\alpha^2 x \tag{67}$$

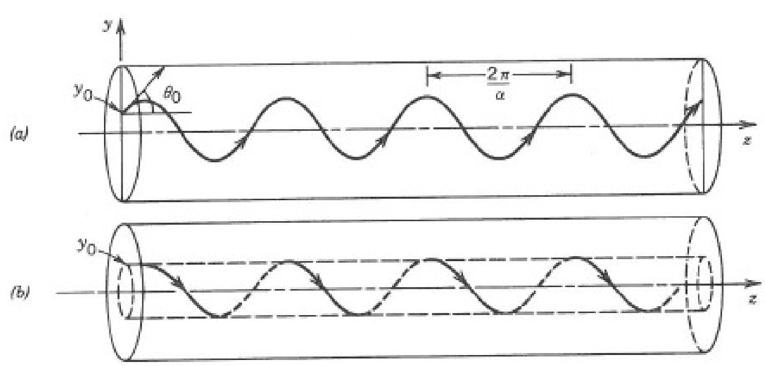
with the solutions

$$y = y_0 \cos\alpha z + \frac{y_0'}{\alpha}\sin\alpha z \tag{68}$$

and

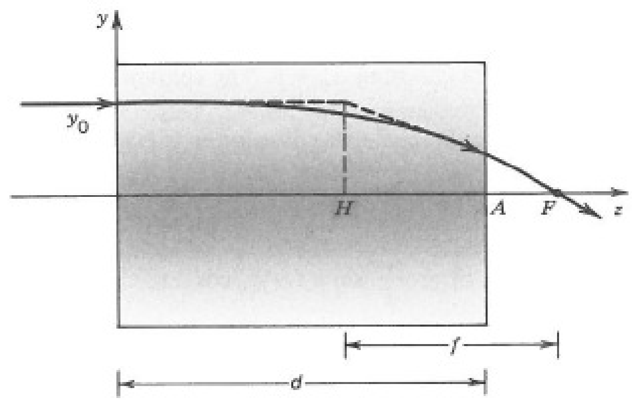
$$x == x_0 \cos\alpha z + \frac{x_0'}{\alpha}\sin\alpha z \tag{69}$$

In general, the solution corresponds to helical ray propagation.



a. Meridional and b. helical rays in GRIN fiber

GRIN Cylinders & Lensing Grin cylinders with parabolic index profile can act as lenses.



SELFOC cylinder used as a lens.

Calculating the focal length will be a homework assignment.

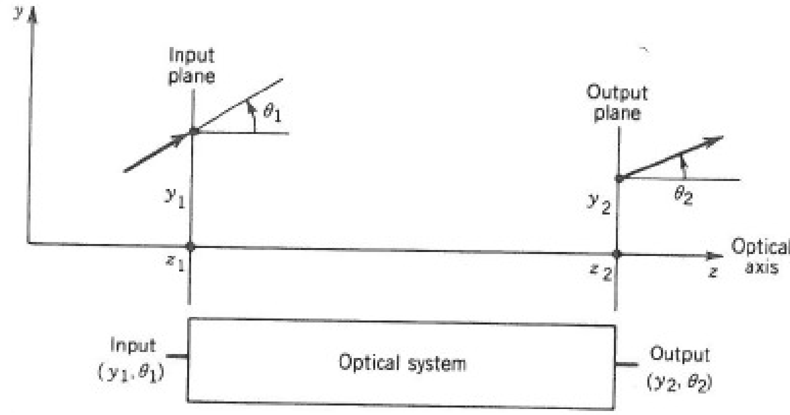
1.7 Matrix Optics

Matrix optics is a way of tracing paraxial rays in a single plane. (Planar geometry, or meridional rays in cylindrically symmetric geometries.)

Rays are described by their position and direction (angle with respect to the optic axis).

In the paraxial approximation, the positions and angles of rays at the entrance and exit surfaces of an optical system are described by two linear equations.

Thus an optical system is described by a 2×2 matrix, called the transfer matrix.



Optical system

The relation betwen the position and orientation of the input ray (y_1, θ_1) and the position and orientation of the output ray (y_2, θ_2) can be written in the form

$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix} \tag{70}$$

where A, B, C and D are real numbers. The matrix

$$M = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \tag{71}$$

is known as the transfer matrix. (Note that if y_2, θ_2, y_1, and θ_1 are given, then we can calculate A, B, C and D. This justifies the assumption of the form of the relationship.)

Important Convention:
We did not worry about signs of quantities when deriving ray equations before. However, now we have to follow convention. The two rules are as follows:
1. If rays are propagating downward, the sign of the angle is negative.
2. The radius of concave concave surfaces (mirrors and lenses) is negative.

Free space propagation We know that in free space,

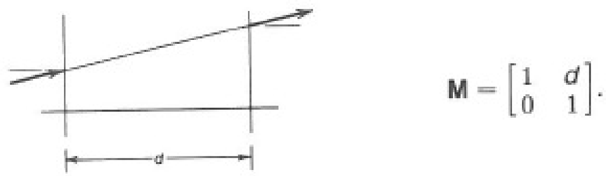
$$y_2 = y_1 + \theta_1 d \tag{72}$$

and

$$\theta_2 = \theta_1 \tag{73}$$

We therefore have for the propagation matrix

$$M = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \tag{74}$$



Refraction at a planar boundary Snell's law, in the paraxial approximation is

$$n_1\theta_1 = n_2\theta_2 \tag{75}$$

and it follows that

$$M = \begin{bmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{bmatrix}$$

Refraction at a spherical boundary Recalling the relation in Eq. 26

$$n_1\theta_1 - n_2\theta_2 = (n_2 - n_1)\frac{y}{R} \tag{76}$$

(note that we are using the sgn convention) we see that, at the interface,

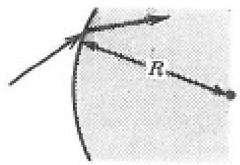
$$y_2 = y_1 \tag{77}$$

and

$$\theta_2 = \frac{n_1}{n_2}\theta_1 - \frac{n_2 - n_1}{R}y_1 \tag{78}$$

and

$$M = \begin{bmatrix} 1 & 0 \\ -\frac{n_2 - n_1}{R} & \frac{n_1}{n_2} \end{bmatrix} \tag{79}$$



Convex, $R > 0$; concave, $R < 0$

$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ -\frac{(n_2 - n_1)}{n_2 R} & \frac{n_1}{n_2} \end{bmatrix}.$$

Transmission through a thin lens Here we use our result from Eq.29

$$\theta_1 - \theta_2 = \frac{y}{f} \tag{80}$$

and we get at once that

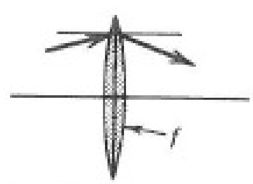
$$y_2 = y_1 \tag{81}$$

and

$$\theta_2 = \theta_1 - \frac{y_1}{f} \tag{82}$$

and

$$M = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \tag{83}$$



Convex, $f > 0$; concave, $f < 0$

$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}.$$

Reflection from a planar mirror Here again we note that

$$y_2 = y_1 \tag{84}$$

and (with the sign convention)

$$\theta_2 = \theta_1 \tag{85}$$

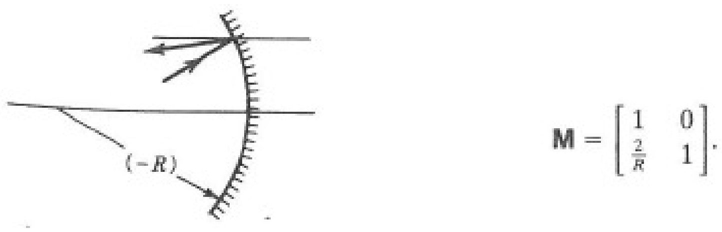
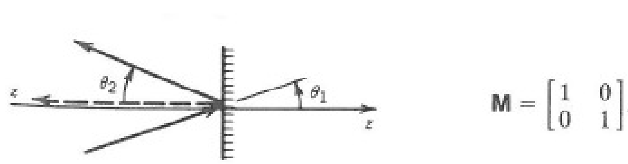


Figure 4:

and the propagatiomn matrix is

$$M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{86}$$



Reflection from a spherical mirror Here again

$$y_2 = y_1 \tag{87}$$

and, from Eq. 18

$$\theta_1 - \theta_2 = 2\theta_0 = -\frac{2y_2}{R} \tag{88}$$

and (again using the previous notation convention,)

$$\theta_2 = \theta_1 + \frac{2}{R} y_2 \tag{89}$$

and the propagation matrix is

$$M = \begin{bmatrix} 1 & 0 \\ \frac{2}{R} & 1 \end{bmatrix} \tag{90}$$

Matrices of cascaded optical components If n optical components are cascaded, the total propagation matrix is just the product:

$$\mathbf{M}_T = \mathbf{M}_n \mathbf{M}_{n-1...} \mathbf{M}_3 \mathbf{M}_2 \mathbf{M}_1 \tag{91}$$

Nb: note the order of the matrices!

Using this approach, many complex optical systems can be analyzed in the geometrical optics approximation.