Chapter 3

Maxwell's Equations

The four equations in the presence of free charges and currents are

$$\nabla \cdot \mathbf{D} = \rho_f \tag{3.1}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{3.2}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{3.3}$$

$$\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t} \tag{3.4}$$

The constitutive equations are

$$\mathbf{D} = \varepsilon \mathbf{E} = \boldsymbol{\varepsilon}_o \mathbf{E} + \mathbf{P} \tag{3.5}$$

$$\mathbf{B} = \mu \mathbf{H} = \mu_o \mathbf{H} + \mu_o \mathbf{M} \tag{3.6}$$

$$\mathbf{J}_f = \sigma \mathbf{E} \tag{3.7}$$

Frequently we will be considering materials which have no free charges, then Maxwell's equations are

$$\nabla \cdot \mathbf{D} = 0 \tag{3.8}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{3.9}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{3.10}$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} \tag{3.11}$$

3.1 The Wave Equation

By taking the curl of Eq. 3.10 we obtain

$$\nabla \times \nabla \times \mathbf{E} = -\frac{\partial}{\partial t}\nabla \times \mathbf{B} \tag{3.12}$$

If μ is independent of position and time and is isotropic, then

$$\nabla \times \nabla \times \mathbf{E} = -\mu\frac{\partial^2}{\partial t^2}\mathbf{D} \tag{3.13}$$

and if ε is independent of time,

$$\nabla \times \nabla \times \mathbf{E} = -\mu\varepsilon\frac{\partial^2 \mathbf{E}}{\partial t^2} \tag{3.14}$$

Finally, making use of the vector identity $\nabla \times \nabla \times \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \boldsymbol{\nabla}^2\mathbf{A}$, we obtain

$$\nabla^2\mathbf{E} - \nabla(\nabla \cdot \mathbf{E}) = \mu\varepsilon\frac{\partial^2 \mathbf{E}}{\partial t^2} \tag{3.15}$$

This equation is central to the work in this course. (Note that we can only derive a similar equation for **H** if the dielectric tensor is isotropic, which is often NOT the case.) Once **E** is known, say by solving Eq. 3.14, **H** can be obtained from Eq. 3.11.