Optics I: Theory CPHY 6/72250
Assignment 5 - Solutions.

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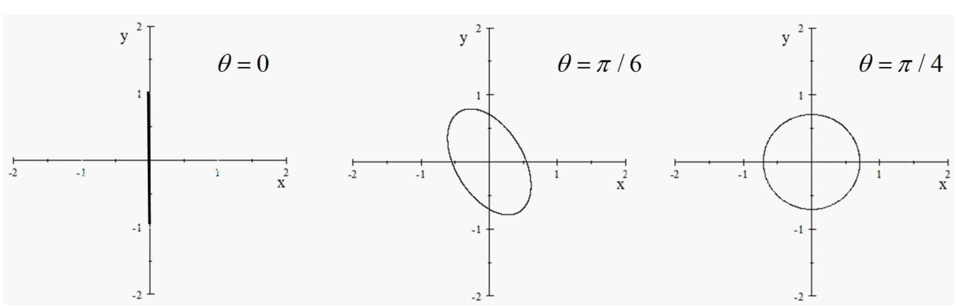
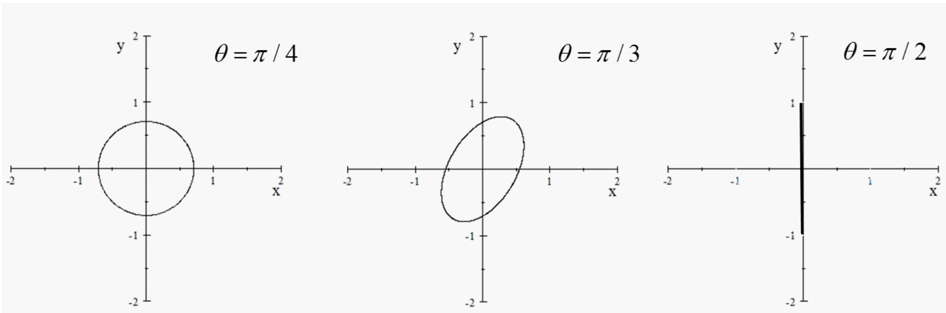
Due: Oct. 29, 2015

1. Monochromatic light propagating along the z-axis shines on a polarizer whose easy axis is along the y-axis. The light transmitted by this polarizer falls on a $\lambda/4$ plate whose slow axis is in the $x-y$ plane, making an angle θ with the x-axis. Draw the polarization ellipse when $\theta = 0$, $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$,. (Show 5 Figures.)

   Initially the light is polarized along the y-direction. One needs to decompose this polarization into one component (of magnitude proportional to $\sin\theta$) along the slow axis, and another (of magnitude proportional to $\cos\theta$) along the fast axis. These are the semi-major and semi-minor axes of the polarization ellipse; as shown below.





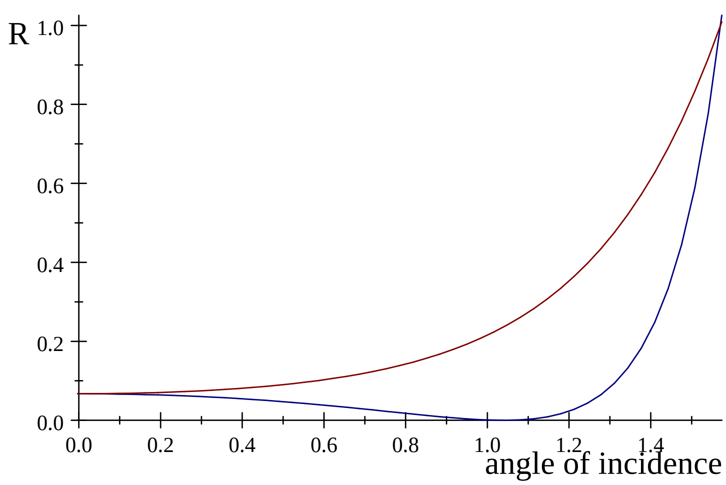
2. Plot R^σ and R^π for an air-glass interface ($n_1 = 1$, and $n_2 = 1.7$) for the cases

   (a) when light is incident from the air side, then

$$R_\pi = \left| \frac{n_2 \cos\theta_i - n_1 \cos\theta_t}{n_2 \cos\theta_i + n_1 \cos\theta_t} \right|^2 = \left| \frac{n_2 \cos\theta_i - n_1 \sqrt{1 - \frac{n_1^2}{n_2^2} \sin^2\theta_i}}{n_2 \cos\theta_i + n_1 \sqrt{1 - \frac{n_1^2}{n_2^2} \sin^2\theta_i}} \right|^2 \tag{1}$$

   and

$$R_\sigma = \left| \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} \right|^2 = \left| \frac{n_1 \cos\theta_i - n_2 \sqrt{1 - \frac{n_1^2}{n_2^2} \sin^2\theta_i}}{n_1 \cos\theta_i + n_2 \sqrt{1 - \frac{n_1^2}{n_2^2} \sin^2\theta_i}} \right|^2 \tag{2}$$



Reflectance vs. angle of incidence.

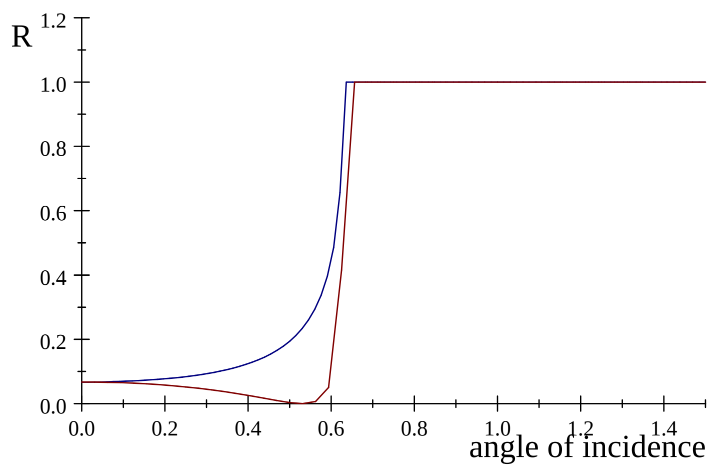
(b) when light is incident from the glass side, then .

$$R_\pi = \left|\frac{n_2\cos\theta_i - n_1\cos\theta_t}{n_2\cos\theta_i + n_1\cos\theta_t}\right|^2 = \left|\frac{n_2\cos\theta_i - n_1\sqrt{1-\frac{n_1^2}{n_2^2}\sin^2\theta_i}}{n_2\cos\theta_i + n_1\sqrt{1-\frac{n_2^2}{n_1^2}\sin^2\theta_i}}\right|^2 \tag{3}$$

and

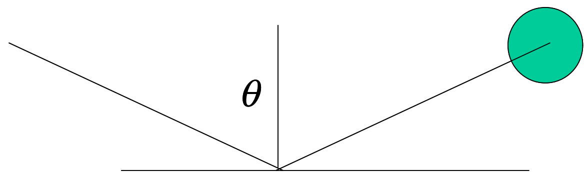
$$R_\sigma = \left|\frac{n_1\cos\theta_i - n_2\cos\theta_t}{n_1\cos\theta_i + n_2\cos\theta_t}\right|^2 = \left|\frac{n_1\cos\theta_i - n_2\sqrt{1-\frac{n_1^2}{n_2^2}\sin^2\theta_i}}{n_1\cos\theta_i + n_2\sqrt{1-\frac{n_1^2}{n_2^2}\sin^2\theta_i}}\right|^2 \tag{4}$$

$n_1 = 1.7, n_2 = 1$



Reflectance vs. angle of incidence

3. As the sun rises over a still pond, an angle will be reached when its image on the water's surface is completely linearly polarized in a plane parallel to the water surface. What is the incident angle?



This must be Brewster's angle for π polarization. Here

$$n_2 \cos\theta_i - n_1 \sqrt{1 - \frac{n_1^2}{n_2^2} \sin^2\theta_i} = 0 \tag{5}$$

or

$$\frac{n_2^2}{n_1^2} \cos^2\theta_i = 1 - \frac{n_1^2}{n_2^2} \sin^2\theta_i \tag{6}$$

which can be rearranged to read

$$\frac{n_2^2}{n_1^2} = \tan^2\theta_i + 1 - \frac{n_1^2}{n_2^2} \tan^2\theta \tag{7}$$

or

$$\tan^2\theta_i = \frac{\frac{n_2^2}{n_1^2} - 1}{1 - \frac{n_1^2}{n_2^2}} = \frac{n_2^2}{n_1^2} \tag{8}$$

or

$$\tan\theta_i = \frac{n_2}{n_1} = \frac{n_{water}}{n_{air}} \simeq \frac{1.33}{1} \tag{9}$$

The angle is therefore $\theta_i = \tan^{-1} n_{water} = \tan^{-1} 1.33 = .926\,09\,\mathrm{rad} = 53.061^\circ$