Optics I: Theory CPHY 6/72250
Assignment 4 Solutions.

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Due: Oct. 25, 2016

1. Read Chapter 7: Polarized Light (http://mpalffy.lci.kent.edu/optics)

2. (10)Red light with wavelength $\lambda = 632.8nm$ and intensity of $1kW/m^2$ is normally incident on a mirror. Calculate the radiation pressure. (Force is the rate of change of momentum; consider the rate of change of linear momentum of the photons.)

   The photon current density is

$$J = \frac{I}{h\nu} \tag{1}$$

   The change of momentum per photon is

$$\Delta p = 2\frac{h}{\lambda} \tag{2}$$

   so the pressure is

$$P = \frac{I}{h\nu}\frac{2h}{\lambda} = \frac{2I}{\nu\lambda} = \frac{2I}{c} \tag{3}$$

   so

$$P = \frac{2\times 10^3}{3\times 10^8}Pa = 6.67\times 10^{-6}Pa \tag{4}$$

3. (10)Circularly polarized green light $\lambda = 532nm$ and intensity $1kW/m^2$ is normally incident on a black piece of paper with dimensions $1cm \times 1cm$. The light is completely absorbed. Calculate the torque on the paper (give magnitude and direction.) (Torque is the rate of change of angular momentum; consider the rate of change of angular momentum of the photons.)
The photon current density is

$$J = \frac{I}{h\nu} \tag{5}$$

The change of momentum per photon is

$$\Delta p = \hbar \tag{6}$$

so the torque density is

$$\frac{torque}{area} = \frac{I}{h\nu}\hbar = \frac{I}{2\pi\nu} = \frac{I\lambda}{2\pi c} \tag{7}$$

so the magnitude is

$$\tau = area \times \frac{I\lambda}{2\pi c} = 10^{-4} \times \frac{10^3 \times 532 \times 10^{-9}}{2\pi \times 3 \times 10^8} = 2.82 \times 10^{-17} N - m \tag{8}$$

The direction of the torque (vector) is along the direction of propagation of light.

4. (20)Consider two identical plane electromagnetic waves propagating in opposite directions.

   (a) (5) Sketch the electric field **E** in space at different times.

      The E field is given by

$$E = E_o \cos(kz - \omega t) + E_o \cos(-kz - \omega t) = 2E_o \cos kz \cos \omega t \qquad (9)$$

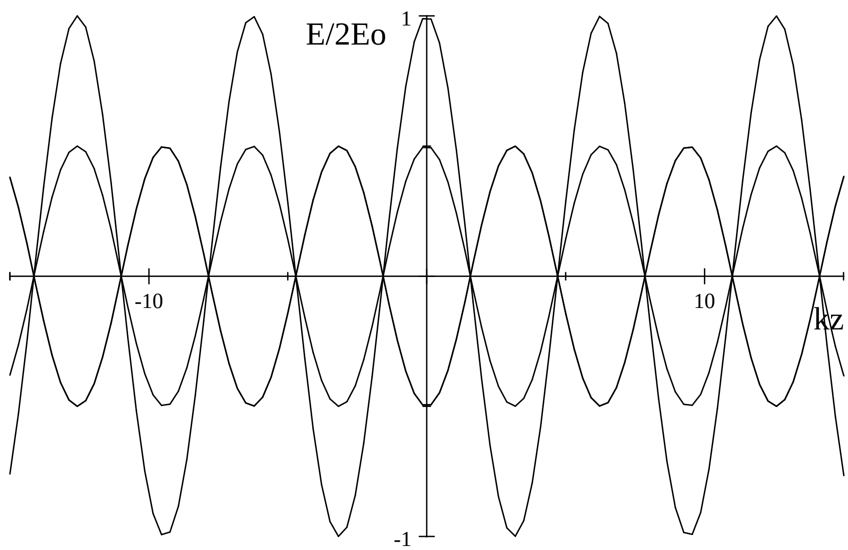

Figure 1: E-field

(b) (5)Sketch the magnetic field **H** in space at different times.

The H field is given by

$$H = \frac{E_o}{Z}\cos(kz - \omega t) - \frac{E_o}{Z}\cos(-kz - \omega t) = \frac{2E_o}{Z}\sin kz \sin \omega t \quad (10)$$

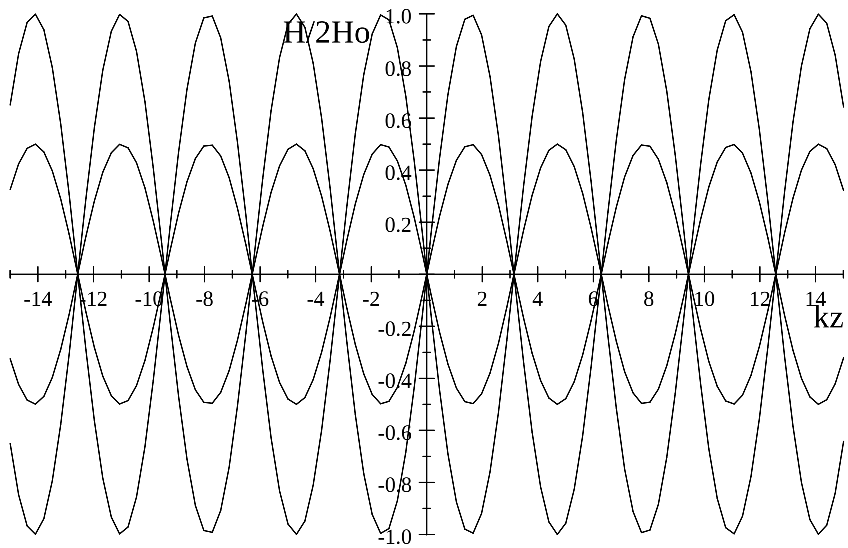

Figure 2: H-field

(c) (5) Give an expression for the Poynting vector $\mathbf{S} = \mathbf{E} \times \mathbf{H}$.

The Polyting vector is

$$\mathbf{P} = 2E_o \cos kz \cos \omega t \hat{\mathbf{x}} \times \frac{2E_o}{Z} \sin kz \sin \omega t \hat{\mathbf{y}} = \frac{E_o^2}{Z} \sin 2kz \sin 2\omega t \hat{\mathbf{k}} \tag{11}$$

(d) (5) Sketch the energy density averaged over time as function of position.

The energy density is

$$\mathcal{E} = \frac{1}{2}\varepsilon E^2 + \frac{1}{2}\mu H^2 = 2\varepsilon E_o^2 \cos^2 kz \cos^2 \omega t + 2\mu \frac{E_o^2}{Z^2} \sin^2 kz \sin^2 \omega t \tag{12}$$

and since

$$Z = \sqrt{\frac{\mu}{\varepsilon}} \tag{13}$$

we have

$$\mathcal{E} = 2\varepsilon E_o^2 \cos^2 kz \cos^2 \omega t + 2\varepsilon E_o^2 \sin^2 kz \sin^2 \omega t \tag{14}$$

and the average energy density is

$$< \mathcal{E} > = \varepsilon E_o^2 \cos^2 kz + \varepsilon E_o^2 \sin^2 kz = \varepsilon E_o^2 \tag{15}$$

so

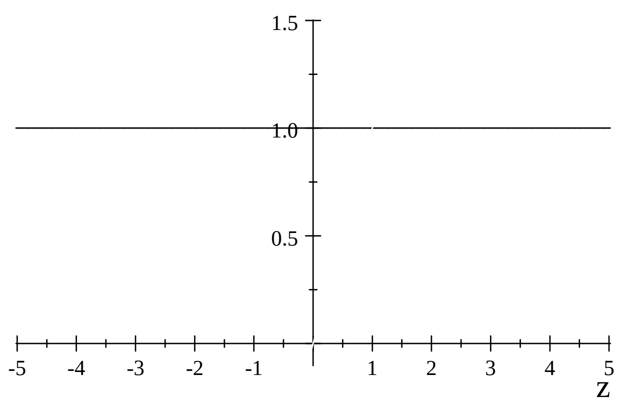

Figure 3: Normalized Average Energy Density