Optics I: Theory CPHY 6/72250
Assignment 2.

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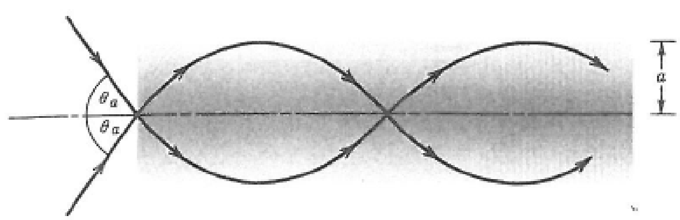
Due: Oct. 5, 2017

1. (8)Consider a graded index fiber with refractive index $n = n_1(1 - \frac{1}{2}\alpha^2 y^2)$ and radius a. A ray is incident from material with index n_0 at its center, making an angle θ_0 with the fiber axis.



Show that, in the paraxial approximation, the numerical aperture is

$$NA = n_0 \sin\theta_0 \approx n_1 \alpha a \tag{1}$$

where θ_0 is the angle for which the ray trajectory is confined within the fiber.

We again start with the ray equation in the paraxial approximation

$$\frac{\partial^2 y}{\partial z^2} = \frac{1}{n}\frac{\partial n}{\partial y} \tag{2}$$

and

$$\frac{\partial^2 y}{\partial z^2} \simeq -\alpha^2 y \tag{3}$$

The solution is

$$y = a \sin \alpha z \tag{4}$$

and the slope at the entrance surface is

$$y' = a\alpha \simeq \theta_1 \tag{5}$$

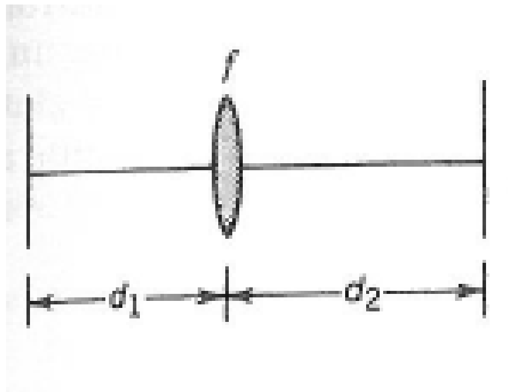
and using Snell's law, we have

$$n_0 \sin \theta_0 = n_1 \sin \theta_1 \tag{6}$$

and approximately

$$NA = n_0 \sin \theta_0 \simeq n_1 \alpha a \tag{7}$$

2. (12)Derive an expression for the transfer matrix of a system comprised of free space/thin lens/free space as shown below.



Show that if the imaging condition

$$\frac{1}{d_1} + \frac{1}{d_2} = \frac{1}{f} \tag{8}$$

is satisfied, all rays originating from a single point y_1 in the input plane reach the output plane at the single point y_2, regardless of their angles. Also show that if $d_2 = f$, all parallel incident rays are focused by the lens onto a single point in the output plane.

We begin by writing down the propagation matrices for all the elements. First, we have propagation in free space through distance d_1 ,

$$M_1 = \begin{bmatrix} 1 & d_1 \\ 0 & 1 \end{bmatrix} \tag{9}$$

then we have transmission through the lens

$$M_2 = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \tag{10}$$

then we have propagation in free space through distance d_2

$$M_3 = \begin{bmatrix} 1 & d_2 \\ 0 & 1 \end{bmatrix} \tag{11}$$

and we have that the total propagation matrix, connecting the output with the input is

$$M_T = M_3 M_2 M_1 \tag{12}$$

or

$$M_T = \begin{bmatrix} 1 & d_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} 1 & d_1 \\ 0 & 1 \end{bmatrix} \tag{13}$$

and

$$M_T = \begin{bmatrix} 1 - \frac{1}{f}d_2 & d_2 - d_1\left(\frac{1}{f}d_2 - 1\right) \\ -\frac{1}{f} & 1 - \frac{1}{f}d_1 \end{bmatrix} \tag{14}$$

If the input ray vector is

$$\begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix} \tag{15}$$

the output is

$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1 - \frac{1}{f}d_2 & d_2 - d_1\left(\frac{1}{f}d_2 - 1\right) \\ -\frac{1}{f} & 1 - \frac{1}{f}d_1 \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix} \tag{16}$$

$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} \theta_1\left(d_2 - d_1\left(\frac{1}{f}d_2 - 1\right)\right) - y_1\left(\frac{1}{f}d_2 - 1\right) \\ -\frac{1}{f}y_1 - \theta_1\left(\frac{1}{f}d_1 - 1\right) \end{bmatrix} \tag{17}$$

: If the imaging condition

$$\frac{1}{d_1} + \frac{1}{d_2} = \frac{1}{f} \tag{18}$$

is satisfied, then

$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} -y_1\frac{d_2}{d_1} \\ -\frac{1}{f}y_1 - \theta_1\frac{d_1}{d_2} \end{bmatrix} \tag{19}$$

That is, all rays originating from (y_1, θ_1) go to through $y_2 = -y_1\frac{d_2}{d_1}$ regardless of direction θ_1.

: If $d_2 = f$, then $d_1 = \infty$, and

$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{1}{d_2}y_1 - \theta_1\frac{d_1}{d_2} \end{bmatrix} \tag{20}$$

If incident rays are parallel to the optic axis, then $\theta_1 = 0$, and

$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{1}{d_2}y_1 \end{bmatrix} \tag{21}$$

and all rays go through the point $y_2 = 0, z = d_2$ regardless of y_1.