Optics I: Theory CPHY 6/72250
Assignment 2.

P. Palffy-Muhoray

Sept. 27, 2015

Due: Oct. 4, 2016

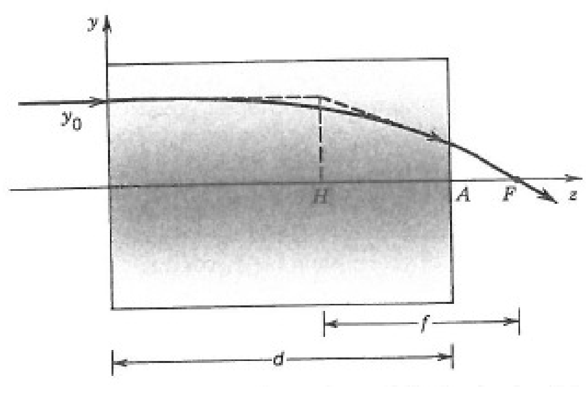
1. (To do this problem, you must have a clear idea what a principal point is; we did not cover this in class. So find out what it is on the web, and once you know what it is, go ahead with the problem. The figure is not very clear, I'm afraid.) Show that a SELFOC slab of length $d < \pi/2\alpha$ and refractive index $n = n_0(1 - \frac{1}{2}\alpha^2 y^2)$ in air acts as a cylindrical lens (a lens with focusing power in the $x - z$ plane) of focal length

$$f = \frac{1}{n_0 \alpha \sin(\alpha d)} \tag{1}$$

Show that the principal point H lies at a distance

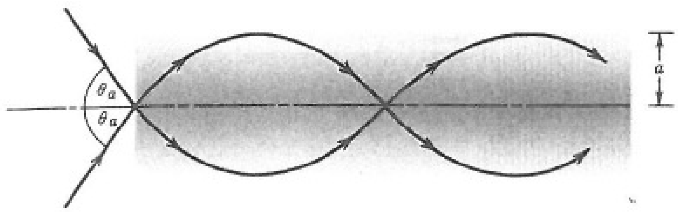
$$AH = \frac{1}{n_0 \alpha} \tan(\frac{\alpha d}{2}) \tag{2}$$

from the slab edge.



Sketch the ray trajetories in the special cases $d = \pi/\alpha$ and $\pi/2\alpha$.

2. Consider a graded index fiber with refractive index $n = n_1(1 - \frac{1}{2}\alpha^2 y^2)$ and radius a. A ray is incident from material with index n_0 at its center, making an angle θ_0 with the fiber axis.

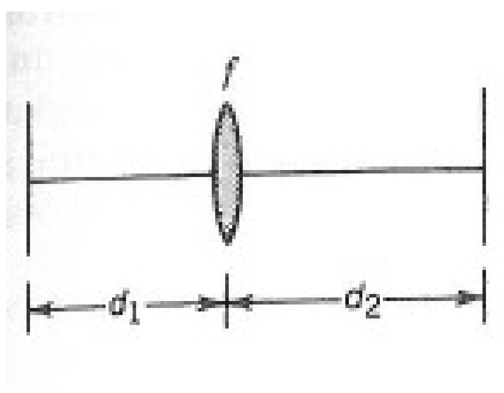


Show that, in the paraxial approximation, the numerical aperture is

$$NA = n_0 \sin\theta_0 \approx n_1 \alpha a \tag{3}$$

where θ_0 is the angle for which the ray trajectory is confined within the fiber.

3. Derive an expression for the transfer matrix of a system comprised of free space/thin lens/free space as shown below.



Show that if the imaging condition

$$\frac{1}{d_1} + \frac{1}{d_2} = \frac{1}{f} \tag{4}$$

is satisfied, all rays originating from a single point y_1 in the input plane reach the output plane at the single point y_2, regardless of their angles. Also show that if $d_2 = f$, all parallel incident rays are focused by the lens onto a single point in the output plane.