

Optics I: Theory CPHY 6/72250

Assignment 1 - Solutions.

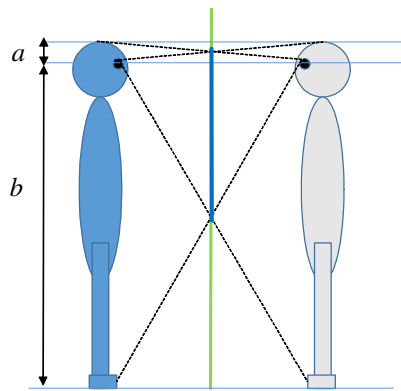
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Due: Sept. 12, 2017

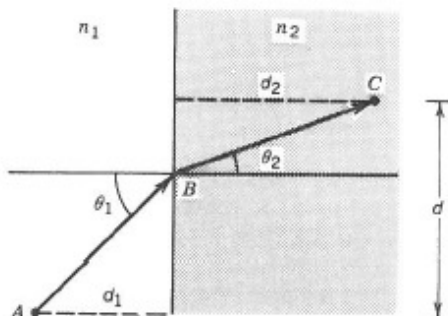
1. What length plane mirror must you purchase to see your full height when it is mounted in a vertical position? (Do not assume that you have eyes on the top of your head.) Make a sketch, and label distances. Consider

the figure shown.



First, we assume that the front of the person is in a plane parallel to the mirror, that is, the points in the object are equidistant from the mirror. We rely on the law of reflection; the angle of incidence is equal to the angle of reflection. Therefore, to see the lowest point in the reflection, the mirror must extend the distance $b/2$ below the level of the eyes, as shown in the figure. Similarly, to see the highest point on the reflection, the mirror must extend the distance $a/2$ above your eyes. So the mirror must have length $(a + b)/2$. Your height is $a + b$. The length of the mirror must therefore be half your height. the result is independent of the object distance; that is, the distance from the mirror.

2. Consider the planar interface shown.



Show that the optical path length from A to C is minimized if

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad (1)$$

This is Snell's Law.

The optical path length OPL travelled by the ray from A to B is just

$$OPL = \frac{n_1 d_1}{\cos \theta_1} + \frac{n_2 d_2}{\cos \theta_2} \quad (2)$$

and

$$d_1 \tan \theta_1 + d_2 \tan \theta_2 = d \quad (3)$$

and d is a constant. Brute force method: We can eliminate one of the angles, say θ_2 . We write

$$\tan \theta_2 = \frac{d}{d_2} - \frac{d_1}{d_2} \tan \theta_1 \quad (4)$$

or

$$1 + \tan^2 \theta_2 = \frac{1}{\cos^2 \theta_2} = 1 + \left(\frac{d}{d_2} - \frac{d_1}{d_2} \tan \theta_1 \right)^2 \quad (5)$$

and so

$$OPL = \frac{n_1 d_1}{\cos \theta_1} + n_2 d_2 \sqrt{1 + \left(\frac{d}{d_2} - \frac{d_1}{d_2} \tan \theta_1 \right)^2} \quad (6)$$

Now minimizing OPL with respect to θ_1 gives

$$0 = n_1 d_1 \frac{\sin \theta_1}{\cos^2 \theta_1} + \frac{1}{2} n_2 d_2 \cos \theta_2 \left[-2 \left(\frac{d}{d_2} - \frac{d_1}{d_2} \tan \theta_1 \right) \left(\frac{d_1}{d_2} \right) \left(\frac{1}{\cos^2 \theta_1} \right) \right] \quad (7)$$

Multiplying through by $\cos^2 \theta_1$ and using Eq. 4,

$$0 = n_1 d_1 \sin \theta_1 - n_2 d_2 \cos \theta_2 \tan \theta_2 \left(\frac{d_1}{d_2} \right) \quad (8)$$

or

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad (9)$$

(This is not an elegant method; it can be done more simply, as many of you have shown. I just did it this way just to show that the method always works.)

3. Consider a series of plane interfaces, all parallel. At the first, the index changes from n_0 to n_1 ; at the second, it changes from n_1 to n_2 ; at the m th from n_{m-1} to n_m . If θ_m is the angle of refraction and θ_{m-1} is the angle of incidence at the m th interface, show that

$$n_0 \sin \theta_0 = n_m \sin \theta_m \quad (10)$$

This is really straightforward; we write Snell's Law in the form

$$\frac{n_0 \sin \theta_0}{n_1 \sin \theta_1} = 1 \quad (11)$$

$$\frac{n_1 \sin \theta_1}{n_2 \sin \theta_2} = 1 \quad (12)$$

$$\frac{n_2 \sin \theta_2}{n_3 \sin \theta_3} = 1 \quad (13)$$

$$\frac{n_{m-1} \sin \theta_{m-1}}{n_m \sin \theta_m} = 1 \quad (14)$$

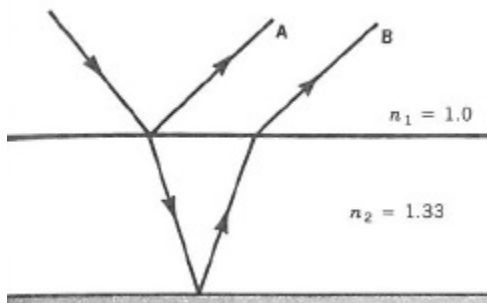
and we multiply all the equations together. Then we get, due to telescoping cancellation,

$$\frac{n_0 \sin \theta_0}{n_m \sin \theta_m} = 1 \quad (15)$$

or

$$n_0 \sin \theta_0 = n_m \sin \theta_m \quad (16)$$

4. A collimated laser beam shines on a tank of water. Part reflects from the surface (beam *A*) and part reflects from the bottom, and exits the water as beam *B*.



Show that the beams are parallel.

We note that the angle of transmission of the ray entering the water is θ_{t1} given by

$$n_0 \sin \theta_i = n_w \sin \theta_{t1} \quad (17)$$

the angle of incidence of the ray striking the water surface from below is also θ_{t1} . So the angle of transmission θ_{t2} of the ray emerging from the water is

$$n_w \sin \theta_{t1} = n_0 \sin \theta_{t2} \quad (18)$$

Combining these, we get

$$\sin \theta_i = \sin \theta_{t2} \quad (19)$$

or

$$\theta_{t2} = \theta_i \quad (20)$$

and since the angle of reflection θ_r is the same as the angle of incidence (Law of Reflection), the two rays leaving the water surface must be parallel.