Optics I Theory CPHY 62250/72250
Assignment 4

Peter Palffy-Muhoray
Due: Nov. 20, 2018.

December 7, 2018

Due: Nov. 20 2018

1. A nematic liquid crystal cell, consisting of two parallel glass plates separated by a distance of $d = 20\mu m$, is oriented so that the plates are in the $x - y$ plane, (the normal to the plates is in the z direction).
   Plane polarized light, polarized along the $(1, 1, 0)$ direction, is normally incident on the cell.
   The nematic director $\hat{\mathbf{n}}$ is in the $(0, 1, 1)$ direction everywhere inside the cell.
   The refractive indices of the liquid crystal are $n_e = 1.7$ and $n_o = 1.4$.
   A polarizer in the $x - y$ plane is placed after the cell.. It can be rotated about the z axis, its orientation is defined by the angle θ, such that when $\theta = 0$, the polarizer transmits light polarized along the $\hat{\mathbf{x}}$, axis.

a. What are the directions of the $\mathbf{D}$ field for the ordinary and extraordinary modes?

The director is in the $y - z$ plane, $\mathbf{k}$ is along the z - axis.

For the ordinary mode, $\mathbf{D}$ is perpendicular to both $\mathbf{k}$ and $\hat{\mathbf{n}}$, it is therefore along the x axis.

For the extraordinary mode, $\mathbf{D}$ is perpendicular to $\mathbf{k}$ and is in the plane containing $\mathbf{k}$ and $\hat{\mathbf{n}}$, therefore it is along the y axis.

b. What are the directions of the corresponding $\mathbf{E}$ fields for the ordinary and extraordinary modes?

For the ordinary mode, $\mathbf{E}$ is parallel to $\mathbf{D}$, and therefore along the x axis.

For the extraordinary mode, $\mathbf{E}$ is in the plane of $\mathbf{D}$ and $\mathbf{k}$, and makes an angle β with $\mathbf{D}$. It is normal to the index ellipsoid.

The angle β is given by

$$\tan\beta = \frac{(n_e^2 - n_o^2)\sin\theta\cos\theta}{n_e^2\cos^2\theta + n_o^2\sin^2\theta}$$

where θ is the angle betwen $\mathbf{k}$ and $\hat{\mathbf{n}}$. In this example, the angle is $\theta = 45^\circ$

c. What are the wave velocities for the ordinary and extraordinary modes?

The refractive index for the ordinary mode is n_o, and the wave velocity is

$$v = \frac{c}{n_o} = \frac{3 \times 10^8}{1.4} = 2.14 \times 10^8 m/s$$

The refractive index for the extraordinary mode is n, given by

$$n = \frac{n_o n_e}{\sqrt{n_e^2 \cos^2\theta + n_o^2 \sin^2\theta}} = 1.528$$

and the vave velocity is

$$v = \frac{c}{n} = \frac{3 \times 10^8}{1.528} = 1.96 \times 10^8 m/s$$

d. What is the phase difference between the two transmitted waves? (ignore reflections)

The ordinary wave, on exit, has the form

$$E_{oexit} = E_{oentrance} e^{ik_o d}$$

and the extraordinary wave, on exit, has the form

$$E_{eexit} = E_{eentrance} e^{ik_e d}$$

so the phase difference is

$$\begin{aligned} \Delta\phi &= (k_e - k_o)d \\ &= \frac{2\pi}{\lambda_o}(n - n_o)d \end{aligned}$$

If the light is green, $\lambda = 514nm$, and

$$\begin{aligned} \Delta\phi &= \frac{2\pi}{514 \times 10^{-9}}(1.528 - 1.4)2 \times 10^{-5} \\ &= (31.295/2\pi)360^\circ = 17697^\circ \\ &= 17697^\circ \bmod 360^\circ = 57^\circ \end{aligned}$$

e. Plot the normalized intensity of light transmitted by the polarizer as function of the angle θ.

Here we will refer to the polarizer angle as θ_p.

The incident light can be decomposed into the two modes.

We then have at $z = 0$,

$$\mathbf{E}_i = E_i \cos 45\,^\circ \hat{\mathbf{x}} e^{i(-\omega t)} + E_i \cos 45\,^\circ \hat{\mathbf{y}} e^{i(-\omega t)}$$

and the transmitted light at the exit surface, at $z = d$, is

$$\begin{aligned}
\mathbf{E}_t &= E_i \cos 45\,^\circ \hat{\mathbf{x}} e^{i(k_9 d - \omega t)} + E_i \cos 45\,^\circ \hat{\mathbf{y}} e^{i(k_e d - \omega t)} \\
&= \frac{E_i}{\sqrt{2}} (\hat{\mathbf{x}} e^{ik_o d} + \hat{\mathbf{y}} e^{ik_e d}) e^{-i\omega t}
\end{aligned}$$

Now we need to know what is transmitted by the polarizer. The polarizer transmits light polarized along its easy axis, let us call this direction $\hat{\mathbf{p}}$, where

$$\hat{\mathbf{p}} = \cos\theta_p \hat{\mathbf{x}} + \sin\theta_p \hat{\mathbf{y}}$$

We can always decompose light into a component parallel to any given direction, and one perpendicular to this. The component of light which is parallel to $\hat{\mathbf{p}}$ will be transmitted.

Light transmitted by the polarizer in this case will have an electric field $\mathbf{E}_p$ given by

$$\mathbf{E}_{tp} = (\mathbf{E}_t \cdot \hat{\mathbf{p}})\hat{\mathbf{p}}$$

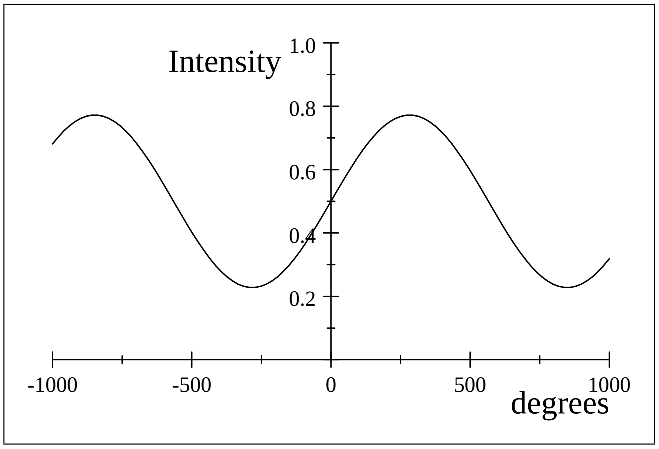
or

$$\mathbf{E}_{tp} = \frac{E_i}{\sqrt{2}} (\cos\theta_p e^{ik_o d} + \sin\theta_p e^{ik_e d}) e^{-i\omega t} \hat{\mathbf{p}}$$

The transmitted intensity is proportional to $|\mathbf{E}_{tp}|^2 = \mathbf{E}_{tp}\mathbf{E}_{tp}^*$, and

$$\begin{aligned}
I &= \frac{1}{2} I_i (\cos\theta_p e^{ik_o d} + \sin\theta_p e^{ik_e d})(\cos\theta_p e^{-ik_o d} + \sin\theta_p e^{-ik_e d}) \\
&= \frac{1}{2} I_i (\cos^2\theta_p + \sin^2\theta_p + \cos\theta_p \sin\theta_p (e^{i(k_o - k_e)d} + e^{-i(k_o - k_e)d}) \\
&= \frac{1}{2} I_i (1 + 2\cos\theta_p \sin\theta_p \cos((k_o - k_e)d)) \\
&= \frac{1}{2} I_i (1 + \sin 2\theta_p \cos(57\,^\circ))
\end{aligned}$$

where I_i is the incident intensity.



f. What is the angle between the wave vector and the Poynting vector inside the cell?

This angle is again β, given by

$$\begin{aligned}\beta &= \tan^{-1}\frac{(n_e^2 - n_o^2)\sin\theta\cos\theta}{n_e^2\cos^2\theta + n_o^2\sin^2\theta} \\ &= (0.189\,45/\pi)180\,^\circ \\ &= 10.9\,^\circ\end{aligned}$$

$(0.189\,45/\pi)180 = 10.\,855$

g. Sketch the index ellipsoid, and show the fields and the wave and Poynting vectors for both ordinary and extraordinary modes.