Optics I Theory CPHY 62250/72250

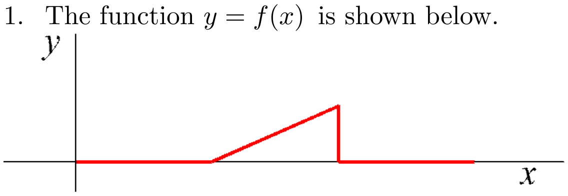
Assignment 1: Solutions

Peter Palffy-Muhoray

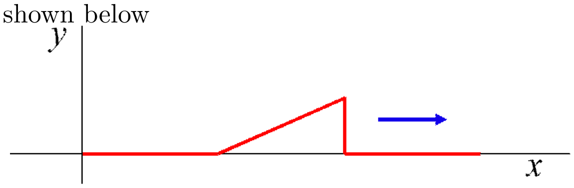
Due: Sept. 16, 2018

September 20, 2018

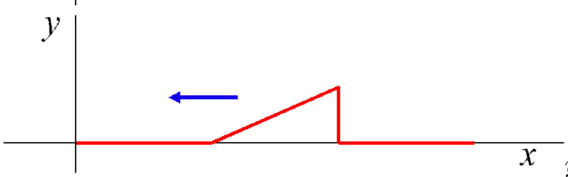
1. The function $y = f(x)$ is shown below.



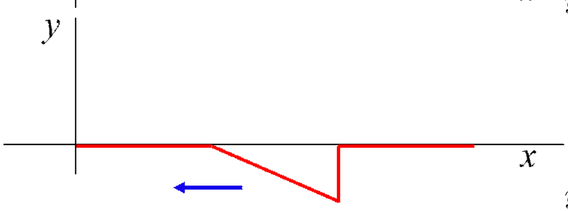
Give the expression for y if y is a travelling wave, travelling with velocity v, as shown below



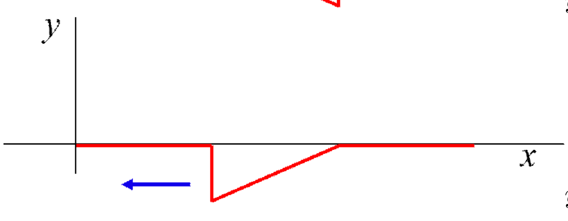
$y = f(x - vt)$



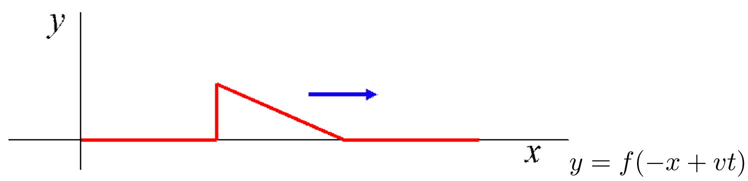
$y = f(x + vt)$



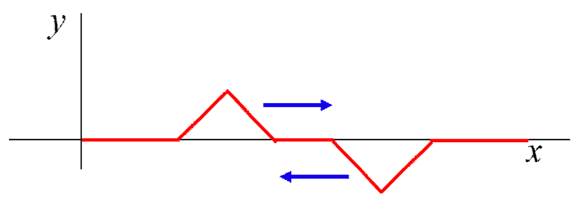
$y = -f(x + vt)$



$y = -f(-x - vt)$



2. A long string has mass density $\rho = 10g/m$ and tension $T = 1N$.
Two triangular waves are travelling in opposite directions on the string as shown below.



The amplitude (maximum vertical displacement) is $1m$, the slope is 45^0.
a. What is the velocity of transverse waves travelling on the string?

The velocity is given by

$$\begin{aligned} v &= \sqrt{\frac{T}{\rho}} \\ &= \sqrt{\frac{1}{1 \times 10^{-2}}} m/s \\ &= 10.0 m/s \end{aligned} \quad (1)$$

b. What is the impedance of the string?

The impedance is given by

$$\begin{aligned} Z &= \sqrt{\rho T} \\ &= 0.10 kg/s \end{aligned} \quad (2)$$

c. What is the kinetic energy of each triangular wave?

The kinetic energy density is

$$K.E./l = \frac{1}{2}\rho(\frac{\partial y}{\partial t})^2 \quad (3)$$

Now

$$\frac{\partial y}{\partial t} = \frac{1m}{0.1s} = 10m/s \tag{4}$$

and

$$\begin{aligned} K.E./l &= 0.5 \times 1 \times 10^{-2} \times 100N \\ &= 0.5N \end{aligned} \tag{5}$$

The total kinetic energy is

$$K.E. = K.E./l \times l = 0.5N \times 2m = 1J \tag{6}$$

d. What is the potential energy of each triangular wave?

The potential energy density is

$$\begin{aligned} P.E./l &= \frac{1}{2}T(\frac{\partial y}{\partial x})^2 \\ &= 0.5 \times 1 \times (1)^2 \\ &= 0.5N \end{aligned} \tag{7}$$

and the total potential energy is

$$P.E. = P.E./l \times l = 0.5N \times 2m = 1J \tag{8}$$

This is the same as the kinetic energy, as expected.

e. Describe, in words, what the string looks like at the instant the triangular waves are on top of each other.

Since the wave equation is linear, the solution for two waves is always just the sum of the two individual waves. Adding the amplitudes of the two waves whent they are on top of each other gives zero. Therefore there is no vertical displacement of the string at this instant, it is perfectly horizontal everywhere.

f. Calculate the total energy in the string at the instant the triangular waves are on top of each other.

Denoting the triangular shape by $f(x)$, we have for the two waves

$$y = f(x - vt) - f(-x - vt) \tag{9}$$

Since $f(x) = f(-x)$, we can write

$$y = f(x - vt) - f(x + vt)$$

at $t = 0$, $y = 0$ for all x.

The potential energy density is

$$P.E./l = \frac{1}{2}T(\frac{\partial y}{\partial x})^2 \tag{10}$$

If $y = 0$, then $\partial y/\partial x = 0$, and therefore the potential energy density everywhere is zero.

The kinetic energy density is

$$K.E./l = \frac{1}{2}\rho(\frac{\partial y}{\partial t})^2 \tag{11}$$

and since $y = f(x - vt) - f(-x - vt)$ we have, in general,

$$\begin{aligned} K.E./l &= \frac{1}{2}\rho(\frac{\partial f(x-vt)}{\partial t} - \frac{f(x+vt)}{\partial t})^2 \\ &= \frac{1}{2}\rho(-v\frac{\partial f(x-vt)}{\partial x} - v\frac{\partial f(x+vt)}{\partial x})^2 \end{aligned} \tag{12}$$

and at $t = 0$, we have

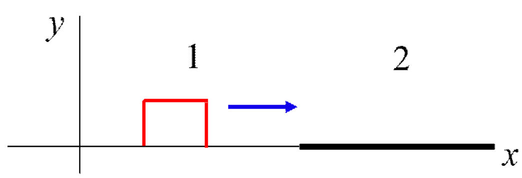
$$K.E./l = 2\rho v^2(\frac{\partial f(x)}{\partial x})^2 \tag{13}$$

Since $\partial f(x)/\partial x = 1$, we have

$$\begin{aligned} K.E./l &= 2 \times 10^{-2} \times 100 \times 1N \\ &= 2N \end{aligned} \tag{14}$$

and since $l = 2$, the total energy in the string is $4J$. This what is expected; each wave have $2J$ of energy each, so the total energy is always $4J$.

3. Two strings are joined together, with $\rho_1 = 10g/m$ and tension $T_1 = 1N$ and $\rho_2 = 60g/m$ and $T_2 = 1.5N$. A square pulse is travelling along string 1 as shown below.



Sketch, in 5 figures, what happens before, during, and after the pulse reaches the interface.

To calculate the reflection coefficient, we calculate the impedances:

$$\begin{aligned} Z_1 &= \sqrt{T_1\rho_1} \\ &= \sqrt{1 \times 10^{-2}}kg/s \\ &= 0.100kg/s \end{aligned} \tag{15}$$

and

$$\begin{aligned} Z_2 &= \sqrt{T_2\rho_2} \\ &= \sqrt{1.5 \times .06} \\ &= 0.300kg/s \end{aligned} \tag{16}$$

The reflection coefficient is

$$\begin{aligned} r &= \frac{Z_1 - Z_2}{Z_1 + Z_2} \\ &= \frac{0.1 - 0.3}{0.1 + 0.3} \\ &= -\frac{1}{5} \end{aligned} \tag{17}$$

The transmission coefficient is

$$\begin{aligned} t &= 1 + r \\ &= 1 - 0.5 \\ &= 0.5 \end{aligned} \tag{18}$$

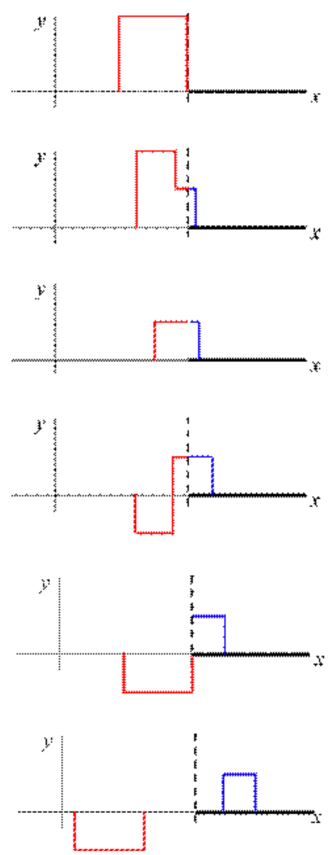
We also calculate the velocities.

$$\begin{aligned} v_1 &= \sqrt{\frac{T_1}{\rho_1}} \\ &= \sqrt{\frac{1}{10^{-2}}} \\ &= 10.0m/s \end{aligned} \tag{19}$$

and

$$\begin{aligned} v_2 &= \sqrt{\frac{T_2}{\rho_2}} \\ &= \sqrt{\frac{1.5}{0.06}} \\ &= 5m/s \end{aligned} \tag{20}$$

We can now draw what happens.



y
x